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AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

PAPERS AND DISCUSSIONS

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THE BUTTRESSED DAM OF UNIFORM STRENGTH

By Herman Schorer,* Assoc. M. Am. Soc. C. E.

Synopsis

Any investigation of the principal stresses in buttressed dams, as designed at present, reveals the fact that the maximum compressive stress occurs only at one single point in each horizontal section. At the same time, the up-stream portion of the buttress may disclose considerable tension in the direction of the second principal stress.

The object of this paper is to present the general theory of a buttressed dam of definitely known and uniform strength—to discuss its dimensional and quantity laws and to make suggestions for incorporating the theoretical findings in practical designs.

The theory is based on the conception of elementary arched column units, which transmit the water pressure from their respective deck areas directly and in uniform compression to the ground, together with the accumulated dead load. The elementary column units can be combined to a monolithic buttress system, in which the first principal stress is uniform compression throughout and the second principal stress is zero.

Secondary stresses due to shrinkage, temperature changes, etc., are largely eliminated by dividing the buttress system into several arched column units of practical dimensions, which units, together with their monolithically connected decks, are completely separated from each other by continuous joints. Climatic influences are further reduced by designing the down-stream column units so as to enclose the structure completely.

The independent column units will also permit the construction of the dam to the ultimate height in independent steps at convenient time intervals. The dangerous and expensive necessity of anchoring new concrete to the older structure is thus successfully avoided.

A buttress system thus designed may be used in connection with any of the common deck types. The method is illustrated in a tentative example for

Note.—Written discussion on this paper will be closed in **March, 1931**, *Proceedings*. The original manuscript of the paper was received by the Secretary on March 12, 1929, and this revised copy on September 26, 1929.

* Hydr. Engr., Thebo, Starr & Anderton, San Francisco, Calif.

a dam 300 ft. in height, using a simple cantilever deck. Diagrams have been prepared, which will enable the designer to determine quickly the most important elements for any desired combination of the main design variables.

Considerable saving in cost can be realized for any height to which a gravity dam with the same apparent factor of safety may be built. With a design stress of 500 lb. per sq. in., the maximum possible height of the buttressed dam of uniform strength is about 800 ft.

Introduction

Hollow dams of the reinforced concrete type generally consist of three distinct members: First, the deck, forming a water-tight up-stream face; second, the buttress, or supporting system, transmitting the combined water and dead load to the foundation; and, third, the foundation, which transmits the total load to the ground.

The design of the foundation structure is governed entirely by local geological conditions which determine the permissible bearing pressures. The following analysis assumes that the dam is founded on solid rock, so that no special foundation structure will be required. The most common deck types are the simple beam slab of the Ambursen dam and the multiple-arch deck. The supporting system of both types generally consists of triangular buttresses with rectangular, horizontal sections.

Any analysis of the relative yardage in deck and buttress reveals the fact that with increasing height the volume of the buttress becomes by far the largest item. The cost of high hollow dams, therefore, is governed largely by an economical design of the supporting system.

The arbitrary, triangular buttress, which is commonly used, has been borrowed directly from the gravity dam. The same may be said of the ordinary buttress analysis, based on the stress determination in horizontal sections, when stability is assumed to be assured if the vertical stress components and the horizontal shearing stresses do not exceed safe limits.

B. F. Jakobsen, M. Am. Soc. C. E., first called attention* to the necessity of determining the principal stresses in the buttresses of multiple-arch dams, because the ordinary analysis may not reveal the existence of large tensile stresses. Later, Messrs. N. Kelen† and Carl Boegh‡ also gave solutions for the principal stresses in wedge-shaped buttresses. The older method of analysis, which considers horizontal sections only, must now be regarded as obsolete and unreliable.

Unfortunately, the principal stress determination does not simplify the design, because the analysis must begin with dimensions already known. For this reason all the present design methods are based on repeated trial analysis, a procedure which is well described§ by Fred A. Noetzli, M. Am. Soc. C. E.

* *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 276.

† "Die Staumauern," by N. Kelen, p. 81, Julius Springer, 1926.

‡ "Beitrag zur Berechnung der Spannungsverteilung in Stuetzpfeilern von Stauwehren aufgeloester Bauweise," by Carl Boegh, *Beton und Eisen*, 1927, p. 331.

§ "The Design and Construction of Dams," by Edward Wegmann, M. Am. Soc. C. E., Eighth Edition, 1927, p. 439.

For economic reasons, and for reasons of safety, the ideal design must always aim at a structure of uniform strength. It is evident that the arbitrary, triangular buttress shape, with rectangular horizontal sections, does not necessarily approach the structural ideal. The trial method also involves a large amount of labor, and the derivation of general dimensional and quantity laws, based on the arbitrary shapes, would lead to a waste of material in higher dams.

In order to eliminate the trial computations the following direct design method for the supporting system is proposed. The water pressure, acting on an elementary deck area, is transmitted to the foundation in the most direct way by elementary arched columns, dimensioned so that the compressive stress is constant throughout. These elementary columns can then be combined to a monolithic buttress of uniform strength. They may also be built as adjacent members, separated from each other by continuous joints, so that secondary stresses due to shrinkage, temperature changes, etc., can practically be eliminated.

The available space does not permit the full presentation of the analysis, and the mathematical expressions, therefore, must be limited to the fundamental results. For the detailed derivations the reader is referred to a more complete manuscript of the paper entitled, "The Hollow Dam of Uniform Strength," filed by the writer in the Engineering Societies Library, New York, N. Y.

Notation

The notation adopted for the discussion of this paper is as follows:

a = a ratio expressing the relation, $\frac{v}{c}$.

A = cross-section area of an elementary column; a subscript, 1, denotes the section at the point, P_1.

b = length of elementary deck area.

c = a ratio expressing the relation, $\frac{f_c}{\gamma_c}$; this unit is a length.

d = horizontal distance, top and bottom, of an arched column at the center line.

D = total height of buttress, identical with maximum depth of water.

e = base of Naperian system of logarithms.

f = unit stress; a subscript, c, denotes compression.

F = a sliding factor; F' is the sliding factor for the buttress of absolute minimum weight.

g = the gravity constant.

h = vertical distance from top of deck to the center of an elementary area of deck slab.

H = horizontal component of force; a subscript, 0, identifies this component with P_0 and a subscript, w, identifies this component with R_w.

i = a ratio expressing the relation, $\frac{D}{c}$, called buttress ratio.

k = a factor signifying the relative deck weights.

m = a variable coefficient that expresses a relation, $\frac{v}{D}$.

P = axial thrust on an arched column; a subscript, 0, denotes water pressure; a subscript, k, denotes top thrust due to water and slab combined; and P'_k denotes top thrust due to water and normal slab component combined.

p = pressure at the top of an elementary arched column; a subscript, v, denotes the vertical component.

r = a coefficient expressing the ratio, $\frac{V_w}{H_w}$.

R = resultant of all forces above the foundation; R_w is the resultant thrust on deck due to water.

S = a force that represents the weight of the elementary deck area; a subscript, B, indicates the component perpendicular to the deck; and a subscript, D, indicates the component transmitted through the slab to the base.

t = thickness of a catenary column; the subscript, 0, indicates a section at any point, N_0, and a subscript, 1, indicates thickness at section N_1, etc.

v = vertical distance from the toe of the deck slab to the center of an elementary area in that slab; Δ_v = horizontal projection of w_0, the width of the elementary area, and $d\,v$ is the horizontal projection of a differential width of deck area.

V = vertical component of force; a subscript, 0, identifies this component with P_0; a subscript, w, identifies this component with R_w; and V'_w = total vertical component of water pressure of buttress of absolute minimum weight.

w = width of an arched column; a subscript, 0, identifies the width with that at N_0; and subscripts, 1, 2, etc., identify corresponding sections at N_1, N_2, etc.

W = weight of structure; a subscript, c, denotes the arched column; a subscript, D, denotes the total deck; and a subscript, B, denotes the total buttress.

x = horizontal distance from the toe of the slab to the center line of the arched column at its base; a subscript, 0, denotes horizontal distance from toe of slab to the center line of the arched column at its top; $d\,x$ is the vertical projection of a differential width of deck area; $\Delta\,x$ = small horizontal interval.

β = angle of R and V.

γ = unit weight; a subscript, c, refers to concrete; a subscript, w, denotes water; and γ'_w is an imaginary unit weight.

λ = weight factor: subscripts, D and B, are for deck and buttress, respectively; λ_0 = factor for buttress with zero deck weight; λ'_0 = factor for buttress of absolute minimum weight; and λ_g = factor for gravity section.

ϕ_0 = angle that deck slab makes with the vertical.

ϕ_1 = angle of column thrust with horizontal at N_1; ϕ_k = angle of combined top thrust, P_k, with horizontal. $\Delta\,\phi_k = \phi_k - \phi_0$; and $\Delta\,\phi$ = angular change of column axis corresponding to $\Delta\,x$.

Arched Column Unit

Referring to Fig. 1 consider an elementary deck area, $A\,B\,C\,D$, and the water pressure, P_0, acting on it. Assume for the present that the weight of the deck is zero. Similar to the thrust of an ordinary bridge arch the thrust, P_0, can be transmitted to the ground by direct compression by the arched column, $N_0\,N_1$. The crown of the arch may be imagined at O, and the right abutment at N_1. The water pressure, P_0, simply replaces the arch thrust at N_0.

If the cross-sectional area of the arch is varied so that the compressive stress is constant at any point, the arch axis will conform to a curve known as the catenary of uniform strength.* The elementary arch, $N_0 N_1$, then is defined as an arched column.

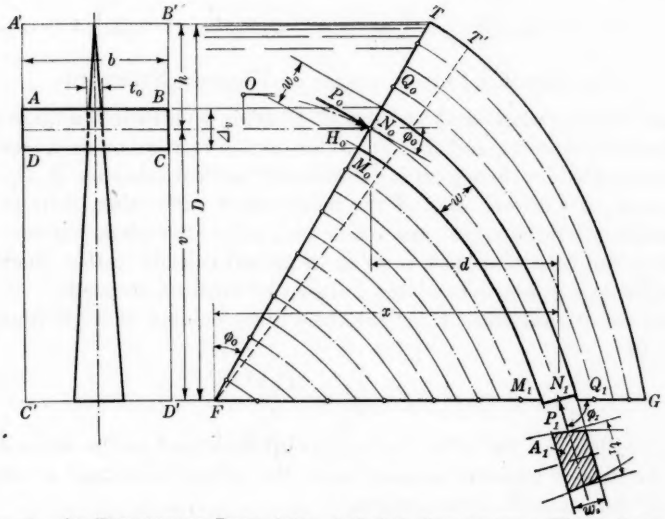

FIG. 1.—AN ELEMENTARY DECK AREA, *A B C D*, SUPPORTING A WATER LOAD, *R*.

Designating by f_c the unit compressive stress and by γ_c the unit weight of the arched column, the expression,

$$c = \frac{f_c}{\gamma_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

represents a parameter, the unit of which is a length. Furthermore, let the ratio, $\frac{v}{c}$, equal a. Designating by P_1 the resultant thrust of the arched column at N_1, and with reference to the dimensions of Fig. 1, the following relations can be derived,

$$\sec \phi_1 = \sec \phi_0 e^a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

or,

$$P_1 = P_0 e^a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The horizontal distance, d, is given by the expression,

$$d = c\ (\phi_1 - \phi_0) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which, the angles, ϕ_1 and ϕ_0, are measured in radians.

Designating by W_c the weight of the arched column between N_0 and N_1, and by H_0 the horizontal component of P_0; then, also,

$$W_c = H_0\ (\tan \phi_1 - \tan \phi_0) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

* "Higher Mathematics," by Mansfield Merriman and Robert S. Woodward, Third Edition, p. 147, John Wiley & Sons, 1902.

The top thickness of the elementary column becomes,

$$t_0 = \frac{b\,\gamma_w\,(D - v)}{f_c} \quad \ldots\ldots(6)$$

Equation (5) can now be written in the form,

$$W_c = b\,\gamma_w\,(D - v)\left[\sqrt{e^{2a}\sec^2\phi_0 - 1} - \tan\phi_0\right]\Delta\,v \quad \ldots\ldots(7)$$

Multiple-Column Buttress of Uniform Strength

Assume that the whole deck strip, $A'\,B'\,C'\,D'$, is divided into a large number of elementary deck areas, and that the water pressure acting on each elementary area is transmitted by corresponding elementary arched columns, $Q_0\,Q_1$, $N_0\,N_1$, etc. If the cross-sectional area of the independent supporting units is shaped so that adjacent column faces just touch each other, the whole buttress can be conceived as built up of a large number of arched column units. Such a buttress is defined as a multiple-column buttress of uniform strength.

The cross-sectional area, A_1, of the elementary column at N_1 is found from Equation (3),

$$A_1 = \frac{P_1}{f_c} = \frac{P_0\,e^a}{f_c} \quad \ldots\ldots(8)$$

If the width, w, of the column at any point is defined as the mean distance between the axis of adjacent column units, the column thickness at any point is definitely determined. Substituting,

$$v = m\,D \quad \ldots\ldots(9)$$

and designating the column thickness at N_1 by t_1, and assuming that $\frac{D}{c} = i$, the following expression results,

$$\frac{t_1}{b} = \frac{D\,\gamma_w\,(1 - m)\,e^{2\,m\,i}\sec^2\phi_0}{c\,\gamma_c\tan\phi_0\sqrt{e^{2\,m\,i}\sec^2\phi_0 - 1} + 1} \quad \ldots\ldots(10)$$

From Equation (4), the horizontal distance, $x = F\,N_1$, and,

$$\frac{x}{D} = m\tan\phi_0 + \frac{1}{i}\,(\phi_1 - \phi_0) \quad \ldots\ldots(11)$$

The angle, ϕ_1, is found from Equation (2); ϕ_1 and ϕ_0 are expressed in radians.

By varying the factor, m, in Equations (10) and (11), the thickness of the buttress along the horizontal plane, $F\,G$ (Fig. 1), can be determined and by repeating the same process for other values of D it is possible to evaluate the shape of the entire buttress.

Equations (10) and (11) indicate that the shape of the buttress is mainly a function of the ratio, $\frac{D}{c}$ (or i), and the angle, ϕ_0. Considering ϕ_0 as a constant, buttresses with the same ratio, $\frac{D}{c}$, have a similar geometrical shape. From Equation (1),

$$\frac{D}{c} = \frac{D}{f_c}\,\gamma_c = i \quad \ldots\ldots(12)$$

This important ratio is called the buttress ratio. Since γ_c is constant for a given case, the ratio, $\frac{D}{c}$, is simply a function of height and stress.

Fig. 2 gives an axonometric view of a monolithic buttress of uniform strength, with horizontal sections at different elevations computed by Equations (10) and (11). In this case, let $c = 200$ ft.; $f_c = 200$ lb. per sq. in.; $\gamma_c = 144$ lb. per. cu. ft.; and $\gamma_w = 62.4$ lb. per cu. ft. It will be noted that for $D = 80$ ft. the horizontal section approaches an isosceles triangle. As the height of the buttress increases the horizontal sections bulge out more and more, so that for $D = 320$ ft. and $D = 400$ ft., the maximum thickness, designated by $t_{max.}$, occurs at a considerable distance from the up-stream face. For $D = 400$ ft., $t_{max.}$ even exceeds the width of the deck strip.

Fig. 2 also shows the buttress ratio for the different values, D, computed by Equation (12) with the given values of γ_c and f_c. Since this ratio determines the shape of the buttress, Fig. 2 also represents the buttress shapes obtained with other unit stresses. Assuming, for instance, $f_c = 400$ lb. per sq. in., $D = 320$ ft., and $\gamma_c = 144$ lb. per sq. in., then the buttress ratio becomes 0.80. (See Equation (12).)

The corresponding buttress is, therefore, identical in shape with that of a buttress with $D = 160$ ft. and $f_c = 200$ lb. per sq. in., as shown.

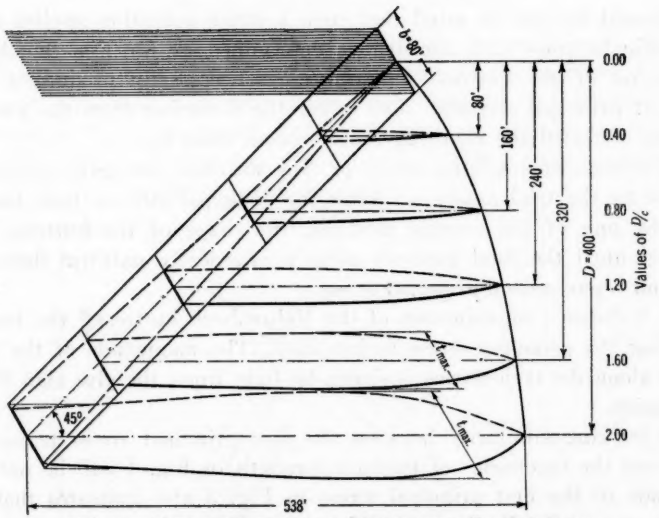

FIG. 2.—DIAGRAM SHOWING A MONOLITHIC BUTTRESS OF UNIFORM STRENGTH.

STRESSES IN THE MONOLITHIC BUTTRESS

The multiple-column buttress (Fig. 1) is built up of independent column units, which can deform freely under the applied load. Since the unit compressive stress is constant, each column will shorten in proportion to its length and the up-stream face moves into the deformed position, FT', which is almost

a straight line. Opposite points along adjacent column faces, therefore, will closely retain their relative position after the deformation has taken place.

Neglecting the influence of Poisson's ratio, the same deformation of the buttress can evidently take place if adjacent column units are connected monolithically with each other, and no shearing forces along the imaginary column faces will be induced. Furthermore, in order to maintain equilibrium, no forces normal to the axis of the catenary columns are required.

Since the buttress is subject mainly to direct stresses in one direction the internal work must approach a possible minimum. From Castigliano's principle of least work it can then be concluded that the stress distribution in the monolithic buttress is very nearly the same as that in the multiple-column buttress composed of independent arched column units.

For practical purposes extreme accuracy is not required, because shrinkage and temperature stresses may easily destroy a theoretically refined stress analysis.

From these considerations the following conclusions are derived:

(1) The catenaries of uniform strength represent principal stress trajectories;
(2) The first principal stress, at any point of the buttress, is a constant compressive stress and equals the design stress, f_c; and,
(3) The second principal stress is zero throughout.

It should be kept in mind that such a stress condition applies only to a monolithic buttress with dimensions determined as for Fig. 2. Any other distribution of the buttress material must necessarily produce a different system of principal stresses. The larger the deviation from the ideal shape, the more distorted the resulting stress system must be.

A buttress, approaching more or less uniform strength, could also be designed by the trial analysis. After the principal stresses have been determined by any of the current methods, the shape of the buttress must be corrected until the final analysis gives a practically uniform first-principal stress and a zero second-principal stress.

Fig. 3 shows a reproduction of the Vöhrenbach buttress,* the heavy lines indicating the principal stress trajectories. The magnitude of the principal stresses along the trajectories is shown by light lines, the plus sign indicating compression.

The striking similarity between the first principal stress trajectories in Fig. 3 and the catenaries of uniform strength in Fig. 1 will be noted. The magnitude of the first principal stress in Fig. 3 also indicates that a more uniform stress distribution could be obtained by decreasing the thickness toward the down-stream face and increasing the thickness toward the lower elevations near the up-stream face. After several trial computations a shape similar to a multiple-column buttress of uniform strength would result. The trial computations would involve considerable labor; the advantage of the direct method is evident.

* "Die Staumauer des Kraftwerkes Vöhrenbach," by Fritz Maier, *Beton und Eisen*, Vol. 23, 1924, p. 13.

Influence of Deck Weight

In analyzing the influence of the deck weight on the shape and weight of the buttress, two extreme cases of deck support are considered.

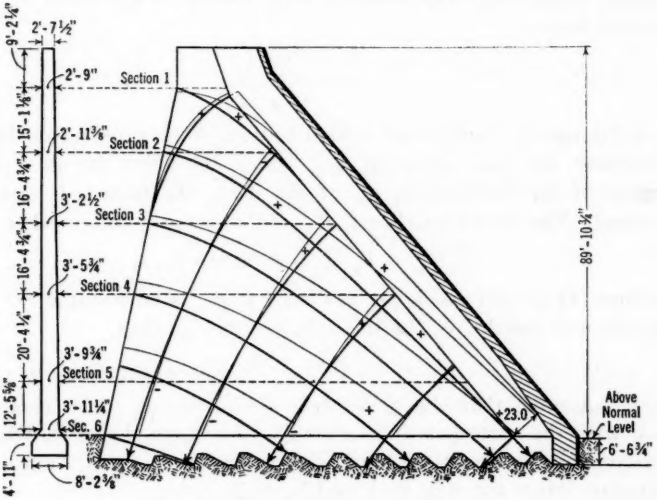

Fig. 3.—The Vohrenbach Buttress.

Case 1.—Assume that the deck is completely separated from the buttress by a frictionless joint along the up-stream face of the buttress, as indicated in Fig. 4. In this case the initial top thrust of the catenary column, designated by P'_k, is composed of the water pressure, P_0, and the normal component, S_B, of the corresponding deck weight, S. The weight component, S_D,

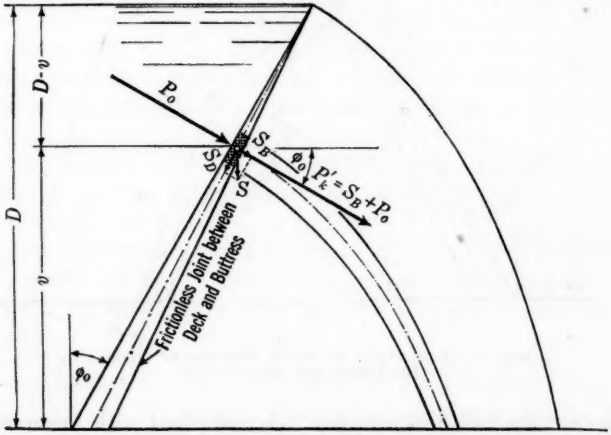

Fig. 4.—Influence of Deck Weight if Deck and Buttress Are Separate.

is transmitted to the foundation through the deck itself. This type of deck support is found in the Ambursen slab deck, except for the uncertainties due

to friction forces, but the assumption of a frictionless joint will be on the safe side.

The weight of the deck is dependent upon the type, working stresses in the deck, span, etc. The relative deck weight is expressed by a factor, k, defined as follows,

$$k = \frac{S}{P_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

For a triangular deck weight distribution, here considered, the factor, k, is constant for any elevation, v. Since its value is also practically independent of the inclination, ϕ_0, of the deck, the factor, k, is called the deck constant. The initial top thrust, P'_k, of the catenary column then becomes,

$$P'_k = P_0 (1 + k \sin \phi_0) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Equations (1) to (12) also apply to Case 1 if P'_k is substituted for R_0, or by replacing the unit weight, γ_w, by the unit weight γ'_w; thus,

$$\gamma'_w = \gamma_w (1 + k \sin \phi_0) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Case 2.—Assume that the elementary deck strip is to be monolithically connected with the corresponding catenary column, as shown in Fig. 5. Adjacent column units are assumed to be completely separated from each other by continuous joints through deck and buttress.

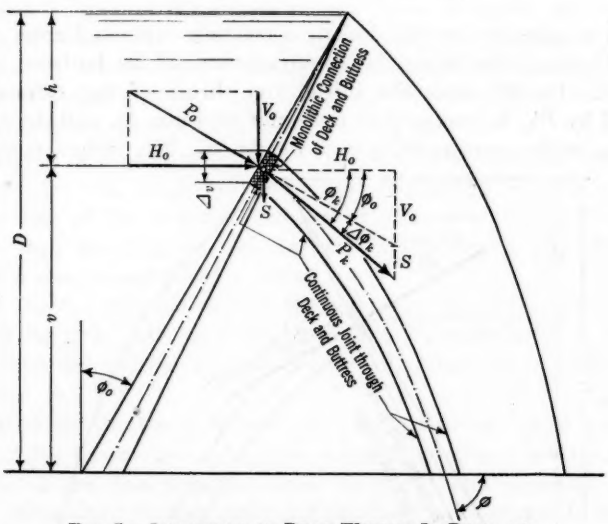

FIG. 5.—INFLUENCE OF DECK WEIGHT IF DECK AND BUTTRESS ARE MONOLITHIC.

In this case the full deck weight, S, is combined with the water pressure, P_0, and the resultant of these two forces, designated by P_k, is inclined at the angle, ϕ_k, to the horizontal. The angle, ϕ_k, is determined from the relation,

$$\tan \phi_k = \tan \phi_0 + k \sec \phi_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

For small values of ϕ_k the following approximate expression, measured in radians, can be derived,

$$\phi_k = \phi_0 + \frac{k \cos \phi_0}{1 + k \sin \phi_0} \quad \ldots\ldots (17)$$

The shape of the buttress, as modified by the deck weight, is given by the following equations, the derivations of which are similar to Equations (10) and (11);

$$\frac{t}{b} = \frac{D \gamma_w (1 - m) e^{2mi} \sec^2 \phi_k}{c \gamma_c \left(\tan \phi_0 \sqrt{e^{2mi} \sec^2 \phi_k - 1} + 1\right)} \quad \ldots\ldots (18)$$

and,

$$\frac{x}{D} = m \tan \phi_0 + \frac{1}{i} (\phi - \phi_k) \quad \ldots\ldots (19)$$

The angle, ϕ, is given by the relation,

$$\cos \phi = \frac{\cos \phi_k}{e^{mi}} \quad \ldots\ldots (20)$$

It may be noted that for $k = 0$, or $\phi_k = \phi_0$, Equations (18) and (19) become identical with Equations (10) and (11) derived under the assumption of a weightless deck.

The weight of the arched column unit is given by the following expression, which is similar to Equation (7),

$$W_c = b \gamma_w (D - v) \left[\sqrt{e^{2m} \sec^2 \phi_k - 1} - \tan \phi_k\right] \Delta v \quad \ldots\ldots (21)$$

Since the angle, ϕ_k, is always larger than ϕ_0, the effect of the deck weight has the tendency to shorten the length of the catenary column. For Case 1 the weight of the column will always be larger than under the assumption of a weightless deck, but this is not necessarily true for the monolithically connected deck. Although the increased top thrust tends to increase the weight of the catenary column, the shortening effect has a tendency to decrease its weight. For small angles, ϕ_0, that is, for steep decks, the shortening effect is very pronounced, whereas the increase of the initial top thrust is comparatively small. The resulting weight of the column then is actually less than that obtained for a weightless deck.

At some intermediate angle, ϕ_0, the two influences balance each other, and for larger angles, ϕ_0, that is, flat decks, the resulting weight of the column becomes greater than for $k = 0$.

In general, it may be stated that for Case 1 the influence of the deck always increases the weight of the buttress. The type described in Case 2 gives a more economical buttress in any case, and, for this reason, becomes of practical importance.

Weight Factor and Sliding Factor Diagrams

The total weight of the buttress is the sum of the weights of all the elementary, arched columns, and is found by integrating Equation (21), which contains the differential equation of the total buttress weight. Since it is not possible to present the integral in explicit form, Equation (21) has been integrated by Simpson's rule.

The main purpose of any dam consists in providing a structure which will resist the horizontal component of the total water pressure. The weight of the structure, therefore, may be expressed conveniently as a factor of the horizontal water pressure. With reference to Fig. 6, the deck constant, k, is then given by the ratio,

$$k = \frac{W_D}{R_w} \quad \ldots\ldots (22)$$

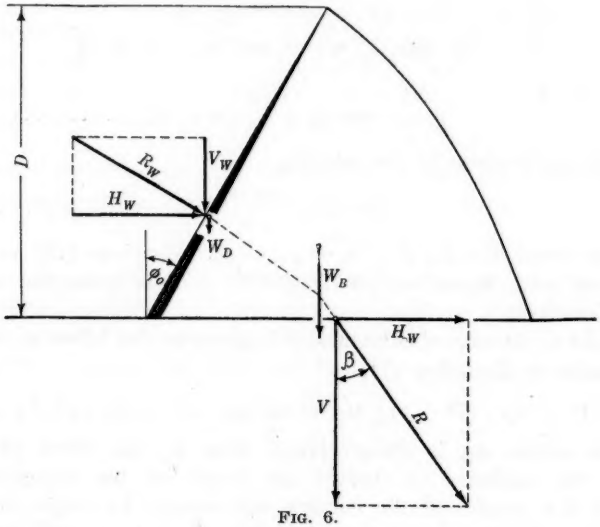

FIG. 6.

The weights of deck, buttress, and combined weights are defined by the following weight factors;

Deck weight factor:

$$\lambda_D = \frac{W_D}{H_w} = k \sec \phi_0 \quad \ldots\ldots (23)$$

Buttress weight factor:

$$\lambda_B = \frac{W_B}{H_w} \quad \ldots\ldots (24)$$

Combined weight factor:

$$\lambda_{BD} = \frac{W_D + W_B}{H_w} = \lambda_D + \lambda_B = k \sec \phi_0 + \lambda_B \quad \ldots\ldots (25)$$

In addition, the direction of the resultant, R, is given by the sliding factor, F,

$$F = \tan \beta = \frac{H_w}{V} = \frac{1}{\tan \phi_0 + \lambda_{BD}} \quad \ldots\ldots (26)$$

The deck weight factor, Equation (23), is a function of the deck constant, k, and the inclination, ϕ_0, of the deck. The buttress weight factor, Equation (24), is a function of k, ϕ_0, and the buttress ratio, i, defined by Equation (12).

The combined weight factor, Equation (25), is also a function of k, ϕ_0, and i, and the same is true for the sliding factor, F, Equation (26).

The weight and sliding factors can be presented in graphical form as functions of the variables, k, ϕ_0, and i, as shown in Figs. 7, 8, and 9.

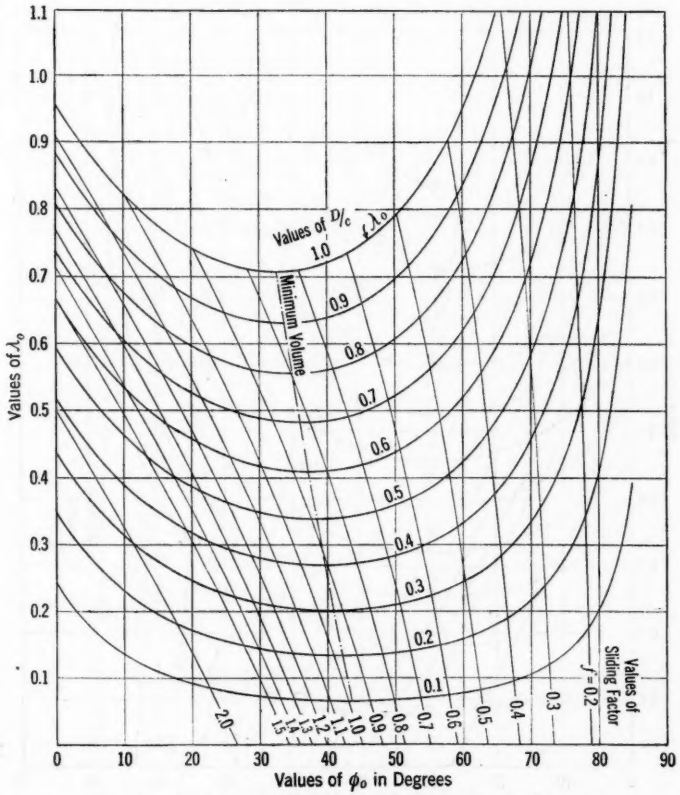

FIG. 7.—GRAPH FOR DETERMINING WEIGHT AND SLIDING FACTORS WHEN $k = 0$.

These diagrams illustrate the quantity laws of the buttressed dam of uniform strength for Case 2. It may be noted that the total volume becomes a minimum for deck inclinations ranging approximately from $\phi_0 = 20$ to $\phi_0 = 40$ degrees. However, the minimum quantities can only be realized in combination with excessively large sliding factors. The deck inclination, therefore, will be governed largely by the limiting sliding-factor values.

The buttress quantities for Case 1 can be obtained from Fig. 7, by computing the horizontal component of the water pressure from Equation (24), with an imaginary unit weight of water as obtained from Equation (15).

The diagrams can be used as guides in the design and for the purpose of preliminary estimates. They are equally applicable to any system of measurements, because the different factors represent pure ratios.

As an example, find the most economical deck inclination as well as the quantities in the deck and buttress of a hollow dam, with the following design data: $D = 200$ ft.; $b = 50$ ft.; $k = 0.10$; $f_c = 400$ lb. per sq. in.; $\gamma_c = 150$ lb. per cu. ft.; $\gamma_w = 62.5$ lb. per cu. ft.; and $F = 0.70$ (maximum permissible value).

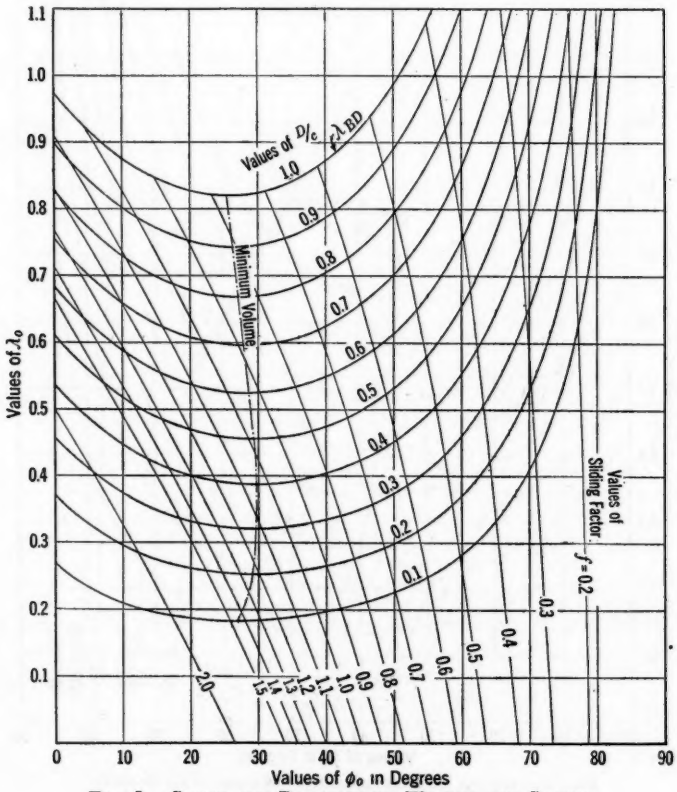
Values of λ_0
Values of ϕ_0 in Degrees
Values of D/c
λ_{BD}
Minimum Volume
Values of Sliding Factor f
0 10 20 30 40 50 60 70 80 90
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1

FIG. 8.—GRAPH FOR DETERMINING WEIGHT AND SLIDING FACTORS WHEN $k = 0.10$

By Equation (12) the buttress ratio is found to be 0.521. Entering Fig. 8, corresponding to the deck constant, $k = 0.10$, it will be noted that, for $\frac{D}{c} = 0.521$, the minimum quantities in the dam require a deck inclination, $\phi_0 = 29$ degrees. The combined weight factor, read at the intersection of $\frac{D}{c} = 0.521$ and $\phi_0 = 29°$, is $\lambda_{BD} = 0.47$. The same intersection gives a sliding factor, $F = 0.97$, which is too large.

In order not to exceed a value, $F = 0.70$, the deck inclination must be limited to $\phi_0 = 43°$; the corresponding combined weight factor is $\lambda_{BD} = 0.50$ which is only 6% larger than the possible minimum.

The total horizontal component of the water pressure, acting on the deck strip under consideration (in pounds), is,

$$H_w = \frac{50 \times 200^2 \times 62.5}{2} = 62\,500\,000$$

With $\lambda_{BD} = 0.50$, the combined weight of deck and buttress, from Equation (25), is, therefore, 50% of H_w. The combined volume, expressed in cubic yards, is,

$$\frac{0.50 \times 62\,500\,000}{150 \times 27} = 7\,720$$

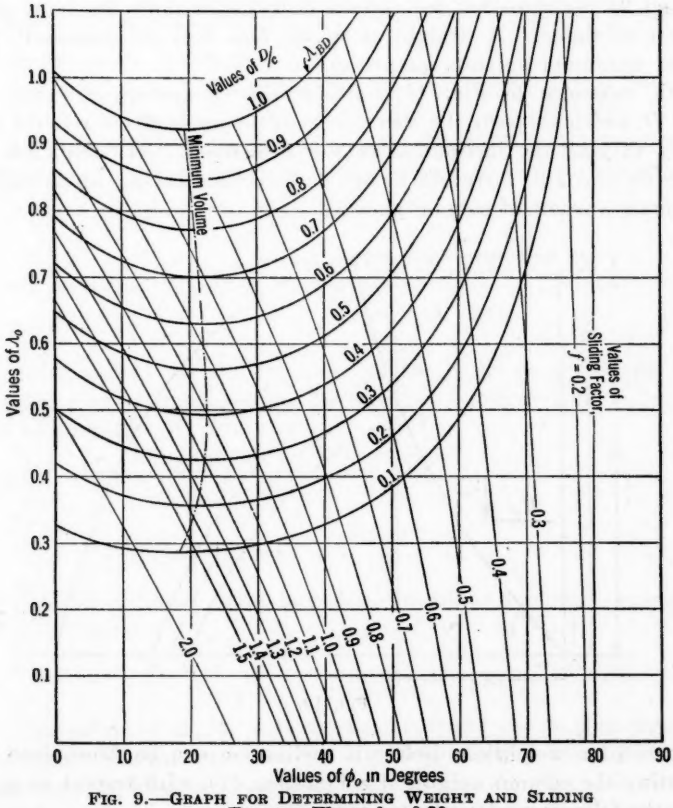

FIG. 9.—GRAPH FOR DETERMINING WEIGHT AND SLIDING FACTORS WHEN $k = 0.20$.

The quantities in the buttress alone are found from Equation (25), which gives the following percentage for the buttress volume,

$$\frac{\lambda_B}{\lambda_{BD}} = \left(1 - \frac{k \sec \phi_0}{\lambda_{BD}}\right) 100 = 72.6$$

These quantities represent the minimum possible volume that can be obtained with the given data. Because of the minimum practical wall thickness the actual quantities will be somewhat larger. If the buttress is provided

with auxiliary stiffening structures, not designed as integral parts of the catenary columns, then the additional volume due to this cause must be added.

Generally, the larger the buttress ratio, $\frac{D}{c}$, the closer the agreement between the actual and the theoretical quantities.

Buttress of Absolute Minimum Weight

The previous deductions have been derived under the assumption that the buttress has a straight up-stream face. The weight factor diagrams (Figs. 7, 8, and 9) indicate that for certain deck inclinations the total quantities become a minimum. A straight up-stream face does not necessarily give the absolute minimum buttress quantities.

With reference to Fig. 10, consider the elementary catenary column, *A B C D*, which supports the water pressure acting over the vertical distance, *dv*. By varying the inclination of the elementary deck strip, *AB*, and by keeping *dv* constant, there will be one deck inclination, ϕ_0, which will require a minimum amount of column material.

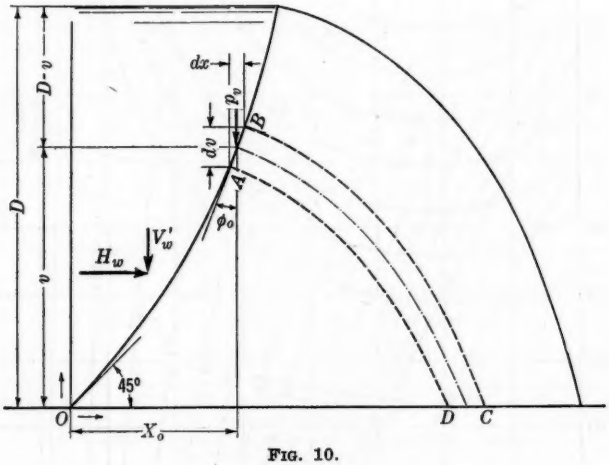

Fig. 10.

In case of a weightless deck this inclination can be determined by differentiating the column weight, W_c, Equation (7), with respect to ϕ_0, which leads to the following relation,

$$\tan \phi = \frac{1}{e^a} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

Equation (27) shows that the most economic deck slope is a function of the elevation, v, and the parameter, c $\left(\text{since } a = \frac{v}{c}\right)$. At the bottom, $v = 0$, the deck inclination is $\phi_0 = 45$ degrees. As v increases the deck becomes steeper and steeper.

Equation (27) also represents the differential equation for the up-stream curve of the buttress, because tan ϕ_0 is identical with the first derivative of x_0 with respect to v. By integration the following relation is obtained:

$$x_0 = c\left(1 - \frac{1}{e^a}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

By substituting $v = mD$, from Equation (9), Equation (28) can be written in the form,

$$\frac{x_0}{D} = \frac{1}{i}\left(1 - \frac{1}{e^{mi}}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Equation (29) shows that the buttress ratio, $\frac{D}{c} = i$, entirely determines the curvature of the up-stream face.

By substituting ϕ_0, from Equation (27), in Equation (7), the minimum weight of the elementary column is obtained. The total weight of the buttress, designated by W'_0, is found by integrating over the total height, D. Designating by λ'_0 the weight factor of the buttress of absolute minimum weight, then,

$$\lambda'_0 = \frac{W'_0}{H_w} = \frac{4}{i^2}(\sin h\, i - i) \quad \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

The buttress weight factor is, therefore, a function of the buttress ratio, i, or $\frac{D}{c}$, only.

In order to find the sliding factor of the buttress of absolute minimum weight, the vertical component of the water pressure, designated by V'_w, must be first determined. By integration,

$$V'_w = r\, H_w \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

in which,

$$r = \frac{2\left(e^{\frac{1}{i}} + i - 1\right)}{i^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

If the sliding factor of the buttress of absolute minimum weight is designated by F', then,

$$F' = \frac{1}{r + \lambda'_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

A comparison with the straight-face buttress shows that the difference between the minimum quantities for $i < 1.0$ is less than 1 per cent. However, the buttress of absolute minimum weight has a sliding factor which is appreciably less than that of the straight-face buttress. The comparison becomes more favorable as the buttress ratio, $\frac{D}{c}$, increases. If the buttress is built with independent column units completely separated from each other by continuous joints along adjacent faces, then the individual bottom columns of Fig. 10 will have a maximum sliding factor value, $F = 1.0$.

With the same quantity of material, the stability of a buttress of absolute minimum weight is superior to that of a straight-face buttress under all conditions.

The influence of the deck, monolithically connected with the corresponding column head, is similar in effect to that derived for a straight up-stream face. The minimum weight of deck and catenary column unit combined is best determined by trial by varying the angle, ϕ_0. If the resulting sliding factor is too large, then the slope of the deck must be decreased until safe sliding-factor values are obtained.

In narrow V-shaped canyons, where the heights of adjacent buttresses change rapidly, the application of the foregoing principles may lead to practical difficulties. Under these conditions the deck of the dam would assume a warped surface, with lateral thrusts acting on the buttress heads.

In case of comparatively wide cross-sections the warping effect is small, so that the foregoing features may be advantageously incorporated in the design of the dam.

Design Example

Fig. 11 represents a tentative design for a buttressed dam of uniform strength, consisting of five catenary column units, completely separated from each other by continuous joints extending through deck and buttress system.

The deck consists of symmetrical cantilever beams or disks, monolithically connected with the column heads and similar in design with an ordinary wall footing. Columns No. 1 and No. 2 are treated as cellular sections, built up of rectangular plates to prevent buckling.

Preliminary computations by Equation (13) indicated an approximate deck weight factor value of from $k = 0.10$ to $k = 0.15$.

With a design stress, $f_c = 400$ lb. per sq. in., and a unit weight, $\gamma_c = 144$ lb. per sq. in., the buttress ratio, i, or $\frac{D}{c}$, becomes 0.75 by Equation (12). With a limiting sliding factor, $F = 0.70$, the inclination of the deck, ϕ_0, is found to be 38° from Fig. 8 ($k = 0.10$). From a similar set of curves, or by interpolation, for $k = 0.15$, the value of ϕ_0 is found to be 36 degrees. The mean value, $\phi_0 = 37°$, was adopted for the design.

The arched column units are designed as follows: First, the water pressure acting on the deck face, and (depending on drainage conditions) also the water pressure acting in the horizontal deck joints, are combined with the weight of the deck to obtain the initial top thrust of each column. For the present purpose the water pressure in the deck joints has been neglected.

The co-ordinates of the column axis and the sectional area at any point can then be computed by Equations (2), (3), and (4). In the present case (Fig. 11), a simple graphical method, based on the differential equation of the catenary of uniform strength, has been used. This method is illustrated in Fig. 12 in which P_0 represents the given top thrust, applied at N_0, and acting at the given angle, ϕ_0, to the horizontal. The horizontal through N_0 is divided into equal intervals, and the verticals, *AB, CD, EF,* etc., are drawn as shown.

The small angle, $\Delta\phi$, expressed in radians, is given by the equation,

$$\Delta\phi = \frac{\Delta x}{f_c}\gamma_c \qquad (34)$$

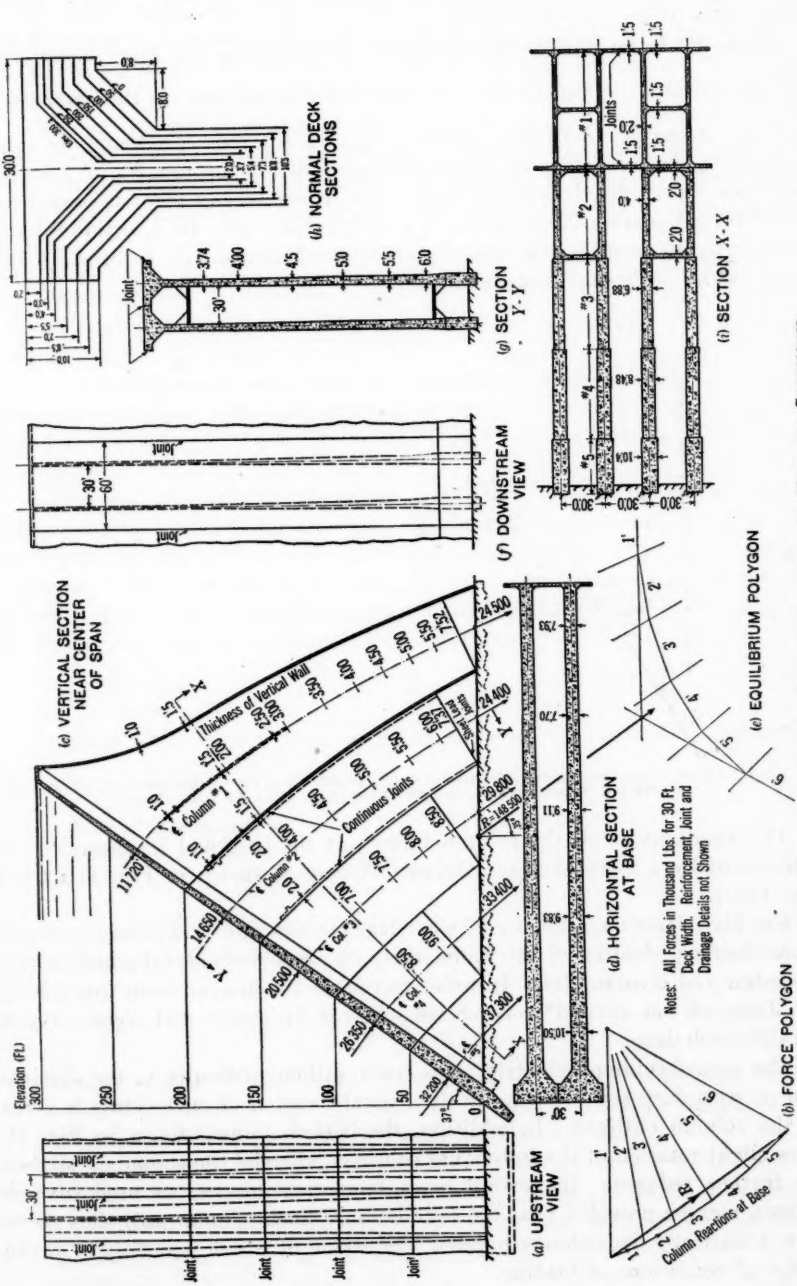
(a) VERTICAL SECTION NEAR CENTER OF SPAN
(b) NORMAL DECK SECTIONS
(c) DOWNSTREAM VIEW
(d) HORIZONTAL SECTION AT BASE
(e) EQUILIBRIUM POLYGON
(f) FORCE POLYGON
(g) SECTION Y-Y
(h) SECTION X-X
(i) UPSTREAM VIEW
Thickness of Vertical Wall
Column #1
Continuous Joints
Joints
Elevation (FL)
Notes: All Forces in Thousand Lbs. for 30 Ft. Deck Width. Reinforcement, Joint and Drainage Details not Shown
Column Reactions at Base

FIG. 11.—DESIGN EXAMPLE OF BUTTRESSED DAM OF UNIFORM STRENGTH.

Assuming $\Delta x = 20$ ft., $f_c = 400$ lb. per sq. in., and $\gamma_c = 144$ lb. per sq. in., then $\Delta\phi = 0.05$.

Since P_0 is tangent at the point, N_0, the construction must begin with $\frac{\Delta\phi}{2}$, as indicated in Fig. 12, and N_1 then is the first point on the catenary of uniform strength. Additional points, N_2, N_3, N_4, etc., are found by equal deflections, $\Delta\phi$, as shown. A continuous curve drawn through N_0, N_1, N_2, N_3, etc., then represents the column axis. The thrust at any point is best determined from Equation (3) by means of a log log slide-rule. Its direction is found graphically by considering that the horizontal component is constant and equal to the horizontal component of P_0.

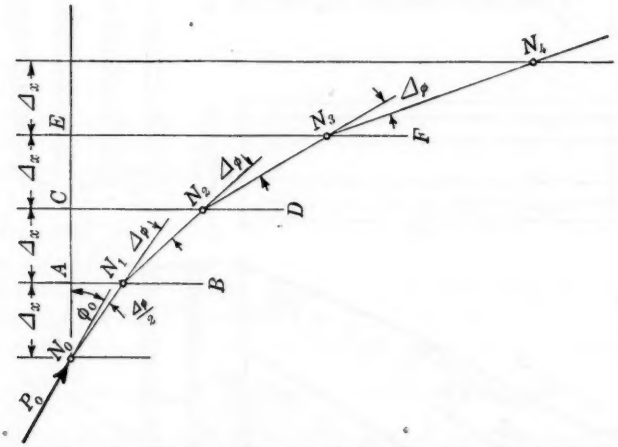

FIG. 12.—GRAPHICAL METHOD OF DETERMINING THE CO-ORDINATES OF THE COLUMN AXIS AND THE SECTIONAL AREA AT ANY POINT.

The resultant of all the column thrusts at the base and its direction are determined by a force and equilibrium polygon, as shown in Fig. 11(*b*) and Fig. 11(*c*).

For high dams the influence of secondary stresses due to shrinkage, temperature changes, yielding of the foundation, etc., deserves careful consideration in design and construction. In order to reduce the danger from this source, Mr. Jakobsen has advised* contraction joints in buttresses and arches of high multiple-arch dams.

The arched-column principle lends itself without difficulty to the elimination of secondary stresses because the logical location of such joints is given by the column outlines. In addition, the bottom joints, shown in Fig. 11, allow slight rotation of the individual columns, whereby the secondary stresses are further reduced. In certain cases temporary hinges, as used in arch bridges, may be provided near the foundation. Each column unit thus represents a statically determinate member, the action of which is definitely known under all conditions of loading.

* *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 276.

Temperature changes in the interior of the dam are reduced to a minimum by the designing column, No. 1 (Fig. 11 (*d*)) so that complete enclosure of the structure is obtained. In severe climates this feature also effectively prevents freezing of the deck from the down-stream side, as reported* by F. W. Scheidenhelm, M. Am. Soc. C. E.

The down-stream face can also be designed as an overflow section, with most of the material acting as a column member. In locations subject to earthquake shocks, all the columns may be designed as adjacent cellular units, whereby it is possible to increase the lateral stability to any desired extent.

Economic Considerations

Buttressed dams are especially adapted for comparatively wide dam sites with solid rock foundations. For this reason the buttressed type is mainly a competitor of the gravity type of dam.

For moderate heights the economy of the Ambursen type has been recognized for some time, and the economic features of the multiple-arch type have been amply discussed by Mr. Noetzli,† V. H. Cochrane,‡ M. Am. Soc. C. E., and others.

In order to evaluate the economic possibilities of the buttressed dam of uniform strength, its maximum theoretical height must be considered. Fig. 2 shows that for the greater heights the thickness of the buttress assumes a maximum value (designated by $t_{max.}$) occurring at some distance from the up-stream face. Since two adjacent buttresses can not occupy the same space, the maximum height of the dam is reached when,

$$\frac{t_{max.}}{b} = 1.0 \qquad (35)$$

Equation (10) shows that the ratio in Equation (35) is mainly a function of the buttress ratio, $\frac{D}{c}$. Designating the maximum height of the dam by $D_{max.}$, then by interpolation,

$$\frac{D_{max.}}{c} = 1.75 \qquad (36)$$

Assuming, for instance, a design stress of $f_c = 500$ lb. per sq. in., and a unit weight of $\gamma_c = 144$ lb. per cu. ft., the maximum theoretical height, in feet, becomes,

$$D_{max.} = 1.75 \times \frac{500 \times 144}{144} = 875$$

The maximum practical height is somewhat lower, due to the deck weight, but it may be concluded that the buttressed dam of uniform strength can be built to considerable heights with ordinary working stresses.

The relative volume of a buttressed dam and that of a gravity dam are best compared by means of the weight factors. The weight factor of a gravity section, designed for full uplift at the up-stream face, with uniform decrease

* *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 772.
† *Loc. cit.*, Vol. LXXXVII (1924), p. 342.
‡ *Loc. cit.*, p. 370.

to zero at the down-stream face, is designated by λ_g. Using the previous designations for the unit weight of water and concrete,

$$\lambda_g = \frac{\gamma_c}{\gamma_w}\sqrt{\frac{\gamma_w}{\gamma_c - \gamma_w}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

With $\gamma_w = 62.5$ lb. per cu. ft. and $\gamma_c = 144$ lb. per cu. ft., $\lambda_g = 2.1$. The weight factor of the gravity section is practically constant for usual heights.

Figs. 7, 8, and 9 indicate that the combined weight factor of the buttressed dam increases about in direct proportion with the buttress ratio, $\frac{D}{c}$, but for usual sliding factors its value is only a fraction of λ_g. For the tentative design of Fig. 11 the combined weight factor value is, $\lambda_{BD} = 0.68$, or about one-third that of a gravity section. Therefore, even assuming that the unit cost of the buttressed dam is twice the unit cost of a gravity section, a saving of about one-third in the concrete cost is realized.

As the height increases the unit cost of the buttressed type decreases, because the cost of deck and formwork represents a smaller and smaller percentage, and the elimination of secondary stresses by means of expansion joints and complete enclosure results in considerable savings of reinforcing steel. Preliminary estimates indicate that the buttressed type of uniform strength will be superior in economy to the gravity dam in almost any case.

The independent column units permit construction to the ultimate height in convenient steps at long-time intervals, without the necessity of anchoring green concrete to the older concrete already in place. The uncertain action, due to different shrinkage and elastic behavior, is thus completely eliminated. At the same time spillway provisions for different construction steps may be conveniently incorporated in the column design.

Since the column units are statically determinate, their stress conditions and factors of safety are definitely known in advance for any contemplated height. To estimate the actual stress condition in the large masses of high gravity dams, even with the provision of vertical contraction planes, is not a simple problem, because successive layers are poured at comparatively slow rates, so that the lower portions may shrink and cool, while the upper portions expand due to chemical heat. It is also evident that the buttressed dam presents more favorable curing conditions because of the possibility of installing effective sprinkling systems, which will not only reduce the shrinkage, but also the setting temperatures.

Considering economy and safety combined it may be stated that the buttressed dam of uniform strength can be built to greater heights than the gravity dam.

Conclusions

From evidence presented in this paper, the writer has come to the following conclusions:

(1).—The arched column principle presents a simple and exact means for designing buttressed dams of uniform strength.

(2).—In case of monolithic buttresses designed by this method the first principal stress is constant compression and the second principal stress is zero.

(3).—Secondary stresses are practically eliminated by building the arched columns as independent, statically determinate units, separated from each other by continuous joints extending through deck and buttress system and complete enclosure of the dam.

(4).—The buttressed dam of uniform strength contains the minimum possible amount of material that can be obtained with a given design stress.

(5).—The exact quantities and design elements can be obtained from diagrams, in which the main variables are expressed as pure ratios.

(6).—The independent arched column units permit construction of the dam in economical steps at long-time intervals—no anchorage between the old and new concrete is required, and spillway facilities may be incorporated in each step with no waste of material for this purpose.

(7).—The buttressed dam of uniform strength is superior in safety and economy to the gravity type for any height to which a concrete dam can be built.

Acknowledgments

The writer deeply appreciates the encouraging assistance and valuable suggestions received during the preparation of the original manuscript from Mikael Juul Hvorslev, Assoc. M. Am. Soc. C. E., Walter Ruppel, Assoc. M. Am. Soc. C. E., and A. C. Schwager, Test Engineer, Pacific Electric Manufacturing Company, San Francisco, Calif.

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PAPERS AND DISCUSSIONS

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ECONOMICAL DESIGN OF HYDRAULIC PIPE-LINE DREDGE

By John Francis Cushing,* M. Am. Soc. C. E.

Synopsis

The hydraulic pipe-line dredge described in this paper† is possibly the largest and most powerful built to date (1930) and embodies some new and novel equipment and construction features. It was designed for the Great Lakes and ocean service as well as for river work. The steel hull is made exceptionally heavy and rigid not only to withstand the machinery vibrations, but to support the dredge without failure if it sinks on an uneven bottom. The curved ends of the hull are the result of experiments to determine that shape which would require the minimum power for towing.

The cutter drive is of unusual flexibility and power, with a specially designed cutter, digging to a depth of 50 ft. The method of control is entirely new.

In order to excavate in a definite area at all times, even in a heavy fog when fixed sights could not be used, a gyroscopic compass is installed which insures accurate and positive operation, as well as an assured return to the site, if the dredge is compelled to seek harbor because of a storm.

The dredge is designed for the maximum economy of capital, labor, and power, and the extensive and thorough investigations by the writer and Mr. R. V. Hoopes to determine the economy of these three elements developed the following conclusions:

(1) That a dredge with a discharge pipe 30 in. in diameter is more economical than that of any smaller size;
(2) That for this size a Diesel-electric installation results in lower production costs than a direct Diesel engine drive; and,
(3) That a single-stage dredge pump driven by the maximum power applicable is most desirable.

Note.—Written discussion on this paper will be closed in **March, 1931**, *Proceedings*.

* Pres., Great Lakes Dredge & Dock Co., Chicago, Ill.

† The Diesel-electric dredge, *New Jersey*, constructed for, and under the specifications of, the Great Lakes Dredge and Dock Company.

Introduction

There has been considerable development in the design and manufacture of dredges of the hydraulic pipe-line type in the last few years. Until about twenty years ago (1910) the use of this type of dredge was limited to the transportation of sand or other loose material through comparatively short distances, after mixing the material in a stream of water through a centrifugal pump. By the adoption of cutting machinery, the field was gradually widened to include more refractory materials, such as clay.

At the same time, the effective range of transportation was extended by improvements in pump design and by an increase in power applied to the pump. This increase was limited for a time to the capacity of reciprocating steam engines with speed characteristics suitable for this class of work.

The application of steam turbines followed the reciprocating engine with very beneficial results.

The more recent improvements of the Diesel type of oil engine has opened a new field of development in the design of dredging equipment, and some installations of Diesel-driven pumps have been made.

The first installation of any considerable size, with Diesel-electric power applied to this service, was made in 1925 by the Port of Portland Commission, Portland, Ore., when the dredge, *Clackamas,* was constructed. It has a pump with a 30-in. discharge, driven by a 2 700-h.p. direct-current motor, the power being developed by four Diesel engines, connected to direct-current generators.

This design and installation had some very special features. The conditions under which the dredge was to operate were not as widely variable as to the nature of material, elevations, distances, exposure, etc., as would be encountered in the field of contracting. The dredge was designed primarily for river operation and it was not necessary to design for lake or ocean conditions.

In general, the design of a dredge of this type for contractors' use involves numerous other considerations which are of vital importance and are not included in a special design for rather fixed conditions. The installation described hereafter, covers a dredge for contractors' use and represents the result of many years of experience, combined with the best engineering and manufacturing skill.

Notation

The following notation has been adopted consistently throughout this paper:

A = cross-sectional area of the discharge pipe, in square feet.

$b = \dfrac{700\,e}{n\,w\,f\,d^{(2-p)}}$, a variable coefficient of the ratio of the cost of labor to the cost of power.

B = a variable coefficient $= \dfrac{700\,e}{n\,w\,f}$.

c = unit cost of labor and power, in dollars per effective pumping hour; c_1 = unit cost of labor; and c_p = unit cost of power used by the main pump.

C = unit cost of labor and power, in dollars per cubic yard.

d = diameter of discharge pipe, in feet.
D = a variable coefficient = $\frac{c_l}{c_p}$.
e = efficiency of pump.
f = coefficient of friction in the pipe.
H = hydraulic head, in feet; H_s = suction head; H_e = elevation head; and H_f = friction head.
L = length of discharge line, in feet.
n = exponent of V in the formula for friction head.
O = output, in cubic yards, per effective pumping hour.
p = exponent of d used in the formula for friction head.
P = horse-power; P_m = water horse-power of the main pump; and P_s = shaft horse-power of the main pump driver = $\frac{P_m}{e}$.
R = rate of discharge, in cubic feet per second.
S = percentage by volume of solid excavated material in the discharge.
V = velocity of discharge, in feet per second; V_e = economic velocity.
w = weight of the mixture of water and solids, in pounds per cubic foot = $(S\ W + U - S)\ 62.5$.
W = weight of dredged material, in pounds per cubic foot.

Economies of Capital, Labor, and Power

Reduced to its fundamentals, the basis for designing a dredge for commercial use must be one of economy—economy of capital, economy of labor, and economy of power. The results of investigation in these three fields will determine the type, size, and capacity of the tool.

Each of these elements is made up of inversely varying factors. The greater the size and capacity, the more refined the type, the greater is the initial investment and, of consequence, the attendant interest and depreciation charges. Against these must be set the decrease in cost of dredging. These items which may be called "fixed charges", are converted into the unit cost of output by dividing the annual fixed charges by the annual capacity. This item of cost varies, therefore, in direct proportion to the initial investment.

The cost of labor required is likewise proportional to the size and refinement of type, and the unit cost of labor is inversely proportional to the capacity. In the same manner, the cost of power varies with the size and type and the unit cost of power is inversely proportional to the capacity.

The hydraulic dredging method is unique as compared to that involving the use of other excavating machines. The dredge excavates its material and immediately transports it to comparatively great distances and there distributes it. The operation is continuous and direct, without the need of intermediate storage of the excavated material common in most other methods of dry or wet excavation. Because of this feature of directness, a fine co-ordination of all the parts or units is essential. Continuous functioning of all the elements or parts is also essential because when one part is out of commission or impaired, the entire dredge is in the same measure out of commission and its ability to do work is impaired. If the cutter is idle, the transportation system, namely, the discharge line, is likewise out of service. If the main pump is out of service, the cutting unit and the transportation unit are likewise out of service. If either the suction line or the discharge line is plugged,

broken, or otherwise idle, the remainder of the dredge is out of commission, etc., each one of the units or parts being highly dependent on all the others.

The total power of the dredge is used largely in cutting or dislodging the material, sucking it up to the pump with a high percentage of water, and there energizing the burdened stream with velocity and pressure to sustain the transportation of the dredged material at a velocity of 10 to 25 ft. per sec. through pipe lines of from 1 000 to 12 000 ft. in length. More than three-fourths of the power is used in overcoming the friction in the pipe line, and this friction varies inversely as about the square of the diameter. Consequently, it is economical to use as large a pipe line as practicable, because the greater the diameter, the less is the friction, and also the range of the dredge. The distance it will transport the dredged material economically increases with the diameter of the pipe line. For the same reason, the use of straight, smooth pipes as far as practical is desirable.

A diameter of 30 in. for the discharge is not necessarily the limit in economy from the standpoint of power consumption per unit of output or from the standpoint of the labor involved in operating the dredge alone. Practically, however, the shore discharge pipe can be handled satisfactorily in diameters up to this size.

The waste water from the enclosed spoil-bank areas can well be taken care of, and the annual capacity of such a dredge is more in keeping with the average demand of a hydraulic dredge than one of a larger size.

The Power Plant

As the power plant of the hydraulic dredge *New Jersey*, represented upward of 25% of the total cost of the dredge, it was a matter given most thorough consideration.

Four two-cycle, air-injection, cross-head type of Diesel engines, each direct-connected to a generator, was the final selection of power plant to fill the many requirements for this dredge in the best way. Among the principal considerations in this design are: (1) The fuel economy of the Diesel engine selected; (2) its convenient and balanced installation in the dredge hull; and (3) the comparatively light weight of the two-cycle engine per unit of power developed in this large size of engine. A four-cycle engine weighing 70 to 80% more would require much higher mean effective cylinder pressures. The cross-head type and comparatively low-speed engines were favorably considered because of their longer life and more reliable service as compared to the trunk type or other high-speed engine.

The range and flexibility of the power were developed by using any combination of the four units, by the convenient pooling of the power, and by its wide-ranged distribution to the different motors on the dredge to meet the almost momentarily changing demands. Particularly is this true in clay work, now quite common, where a wide range of quality of material is encountered frequently in a single swing of the dredge. These alluvial clay deposits, bordering and on the bottoms of rivers and lakes, are mixtures of solid material and water. The solid material is similar to that of shale and slate, which have developed from the clay and mud particles in geological times.

Physical Characteristics of Clay and Shales

Shales probably are produced from the soft clays and mud. They weigh from 160 to 175 lb. per cu. ft., and slates, which have a certain small percentage of porosity, weigh 175 lb. per cu. ft. Considering the influence of heat, and also that the hardest clay particles contain some water, 165 lb. per cu. ft. seems the most probable weight of the solid particles in very soft clay and mud, and this is equal to the weight of most stones. The specific gravity of these solid clay particles is 2.64, assuming the specific gravity of water as 1, and its weight as 62½ lb. per cu. ft. If v is the volume of water in a cubic foot of clay or mud then $(1 - v)$ is the volume of solids, and the weight of the clay per cubic foot will be, $w = 62.5\,v + (1 - v)\,165$. If, for example, the clay is 10% water by volume then the weight would be 156 lb. per cu. ft. If the clay contains 80% of water, the weight, w, equals 83 lb. This shows the wide range of 73 lb., or about 90% variation in the weights of clays commonly encountered in reclamation work.

These values illustrate why clays and muds are more difficult to predetermine and classify and also why they are relatively so costly to handle. Sands or other similar materials do not retain permanently this high content of water and hence are more uniform in weight and other characteristics. In the hydraulic dredging operation, the water content of clays may be changed or it may not be changed substantially, depending on the character of the clay. If it is changed, it will probably return to its original condition after it has been placed in the fill area. In pumping clay the liquid mixture of the suspended particles encountered does not weigh 160 to 175 lb. per cu. ft. as in the case of sands, gravels, and other similar materials; but while these particles rather weigh less, depending on the normal, natural content of water in the clay, nevertheless the weight per cubic foot of clay is greater and the work required to move it from the cut to the fill is greater. This accounts for some of the difficulties of retaining clay fills within the confines of the filled area, because it is manifest that, for higher water contents, the clay is lighter and more fluid. Furthermore, if the fill is a deep one the problem of the compression of clay is involved. When clay is compressed so that the water oozes out, the solid particles remain the same; but the height or depth of the fill is decreased, and it is important to note that the subsidence in the clay fill is not exactly in a direct proportion to the decrease in water content. The subsidence is at an increasing rate. For example, if a clay with 50% of water is compressed to 40% water, the subsidence or decrease in h, the height, would be $\frac{(0.50 - 0.40)}{1 - 0.40}\,h = \frac{1}{6}\,h$. Therefore, it will be seen that if the water content in the fill is to be reduced 10% from that found in the cut, 17% more cubic yards of clay must be pumped from the borrow area than will be paid for in the filled area as commonly measured.

In 1924 and 1925, the writer had an experience with a clay fill on the lake front at Chicago, Ill., in which the loss was in excess of 10% in the pay units of output for the dredge. This resulted largely from such compression of clay. The reclaimed or filled area was approximately 3 000 000 sq. ft., quite

regular in shape, and would correspond to a circular area about 1 960 ft. in diameter, requiring about 110 000 cu. yd. for each vertical foot of fill over the area. The average depth of water before the fill was deposited was about 16 ft. The fill was of the same material as that of the ground below, and when hydraulically deposited, it could be considered as of the same quality after its natural compression. The clay was soft, with a natural slope of between 4 on 1 and 5 on 1 near the surface of the bed of the lake in the cut and in the fill, the lower strata in the cut naturally becoming a little firmer. The dredging was done during two years, settlement in each case continuing over the winter periods when the dredging operation could not be continued because of the exposure to lake storms and ice conditions. Successive surveys were taken over the cut and fill areas.

TABLE 1.—Compression in Clay Fill.

Year.	Average height of fill, in feet.	Quantities, in Cubic Yards.			Percentage of compression.
		Cut.	Fill.	Discrepancy.	
1924.....	18.2	2 030 750	2 002 590	28 160	1.39
1925.....	9.9	1 459 250	1 086 403	372 847	25.5
Total....	28.1	3 490 000	3 088 993	401 007	11.5

Table 1 shows the progress by years. In the placing of the first two-thirds or approximately 18 vertical ft. of the fill, a compression of only 1.39% was produced; however, during the second year in the placing of the remaining one-third (corresponding to a layer about 13 ft. over the previously placed fill) an average of less than 10 vertical ft. was actually found by survey, which constitutes an apparent compression of about 25 per cent. The height of this fill is only about 1½% of the width, which is a comparatively thin layer. It is conclusive, of course, that much of the compression took place in the lake bed and the first 18 ft. of fill above it.

The average apparent settlement on the fill was 3.65 ft. The water content of the clay was approximately 45%, and its weight was 119. lb. per cu. ft., producing an average pressure on the lake bottom of about 1.7 tons per sq. ft. By quite an extensive and careful analysis of this lake-bed clay and by computing the compression, it was possible to account for 266 000 cu. yd., or about 66% of the total discrepancy between the cut and fill measurements given in Table 1.

The subsidence that would result in a compression of 66% is equal to 2.32 ft. in vertical height. This corresponds to a 4% reduction in the water content, so that if clay with 49% water is compressed to 45%, the subsidence would be, $31.75 \left(\frac{0.49 - 0.45}{1 - 0.45}\right) = 2.31$.

The additional power required to handle medium or hard clays is cumulative and places a comparatively great demand on the power plant of the dredge.

More power is required for cutting, much more power for transporting the larger pieces of solids through the pipe, and considerably more power is required for handling throughout this denser and heavier material. More man-power is required on the fill for the distribution of the material. The high velocity required in the pipe line to transport these larger pieces of clay is quickly lost at the end of the discharge pipe. The clay lumps are immediately dropped and pile up near the end of the pipe, and this requires much additional pipe laying for distributing the fill, but, more important, it causes delays and lost working time of the dredge while the pipe is extended or the discharge is switched to another line.

To reduce delays to the dredge, due to these shore operations, a Y-valve that was developed by the writer is being used in the shore line for shifting the discharge from one line to another, without interrupting the operation of the dredge. This valve is a simple device, consisting of a spoon-shaped blade revolving on the axis of the pipe that enters the Y from the dredge so that material is deflected from one branch of the Y to the other. The sealing edges are built so that they will clear themselves, and will not jam, due to the heterogeneous burden in the stream. While the valve is in operation the cross-sectional area (and, therefore, the velocity of the discharge through it) is constant while the discharge is being shifted.

If the fill area is large, the natural slope may vary with the water ratio, and it is very expensive, if not impractical, to place the fill so that it differs much from its natural slope, which varies with the softness or water-ratio in the clay or mud. Clay with 40% of water (weight, 124 lb. per cu. ft.) is considered soft. This water-ratio, or porosity, is like that of average sand (35 to 45%) and the material is nearly of the same weight per cubic foot as sand and water. The natural slope of sand in water is about 25°, but the more fatty clay particles, lacking the sharpness of sand, will only have a natural slope of about 20 degrees. Only the best clays, when thoroughly wet, will reach a slope of 25 degrees. Mud or silt containing 60% of water (104 lb. per cu. ft.), of old formation in lakes, has a natural slope of about 4°, and in newly-formed river mud the slope is probably even less. Clays of different solid matter may vary a little with the same water-ratio, but in clays of the same composition, the slope could be considered as decreasing approximately in a ratio that varies directly as the increase of water percentage.

Such alluvial clays range from very soft to very hard, and frequently contain boulders. Some clays are easy to pump, and others are difficult, developing a high friction coefficient. Some of these clays are broken up into small pieces by the pump, and others are tough and pass through the entire discharge line unbroken or in approximately the same sizes as sliced from the bank by the cutter. It is highly desirable to feed excavated material or burden into the pump and the line stream uniformly so as to reduce the shock to the machinery; and it is more important to reduce the delay due to "slugging" or plugging the suction, pump, or discharge line; but under the conditions described it is very difficult to approach uniformity in cutting or feeding the material into the dredge. In operation a practical limit is reached by working to a margin so that a momentary overload of burden can be passed through

without slugging. The ability to shift power and speed quickly between the swinging motor, cutter motor, ladder hoist motor, and main pump motor is of great advantage. It permits giving to the temporarily overloaded unit a "shot" of power and to the overloading unit a reduction at the propitious moment.

In order to study the problem of designing a dredge it is necessary to have an understanding of the physical properties of materials encountered and the field operating conditions on the dredge and on the fill. Furthermore, it is necessary to know something of the effect of storms, winds, tides, low temperatures, and slides in the cuts and fills.

Preliminary Considerations

In any undertaking, the development of a plan or program for a short period of a few days is simple; such a program for a year is more involved; and for a 5-year period it is much more difficult, as many new problems are thereby brought forward for consideration, to be weighed and fitted into the program in a fashion that will make the entire project feasible and most desirable. These longer programs are necessarily subject to modification from time to time and this must be provided for, but it will be found that the practice of looking many years ahead will result in much more stable and economic progress than otherwise could have been possible. The plans and specifications for a dredge of the character described in this paper must cover a program for a period of 30 to 35 years, or the anticipated life of the dredge.

The application and distribution of power in the Diesel-electric installation are so flexible that the increased fixed charges on the additional electrical equipment and the loss of power in the double conversion are more than offset by the reduction in cost of output.

In a direct Diesel-engine installation it is necessary to provide maximum power capacity for both the main pump and the cutter and other auxiliaries independently. When the full cutter power, however, is not required, the engine supplying this power is at part load and any surplus is not available for other purposes. Similarly, when the full power is not required on the main pump, its engine is operating at part load and the surplus from this source is also not available for other purposes.

Practically, this means that when handling refractory materials, such as hard clay, the power requirement for the cutter is at a maximum while that of the main pump may be reduced, due to a decrease in the percentage of solids in the outflow and possibly to the fact that there may be a lower friction coefficient than with more pliable material. In this case, a proper and efficient distribution of power can be made by motors to all units while drawing current from a common pool of power.

Both the main pump and cutter requirements call for maximum horsepower over a considerable variation of speed. This cannot be accomplished except to a limited extent, with either a reciprocating engine or a turbine; but it can be taken care of almost automatically with motor drive. For instance, in this design, the main pump motor will deliver full-rated horse-power from 250 to 360 rev. per min., without over-heating; the cutter motor will deliver

full-rated horse-power from 250 to 500 rev. per min. at full load with armatures in parallel, and from 125 to 250 rev. per min., at full load with armatures in series. The slow-speed range of the cutter is required only in very soft material so that power supply is ample for that condition.

The operating economy as between labor and power depends, in general, upon the velocity of discharge; that is, between the limits of a minimum velocity required to transport the dredged material and the maximum which can be provided by the pump.

Assuming any fixed set of conditions as to length of line, elevation, total head, percentage of working time, and percentage of solids in the stream, the daily output will be directly proportional to the velocity of the discharge. The power required for pumping will be proportional to the velocity and the total head. The total head consists of the suction head, the static or elevation head, the friction head, and the velocity head—the last being of such a low order as to be negligible in the computation.

For comparison, the suction head and the static head may be assumed as constant for any comparable condition. The friction head is an exponential function of the velocity and the diameter of the pipe. It may be demonstrated mathematically that there is a point of maximum economy between labor and power, a velocity at which the combined cost of labor and power is a minimum. By definition, the total cost for labor and power will be,

$$C = \frac{c_1 + c_p P_s}{0} \quad \ldots\ldots (1)$$

The horse-power required to move a given quantity of liquid against a given head is equal to $\frac{w R H}{550}$, in which, w is the weight of the liquid per cubic foot; R, the rate of discharge, in cubic feet per second; and H, the head, in feet. The shaft horse-power of the driver is this value divided by the efficiency of the pump, or $\frac{w R H}{550 e}$. Furthermore, R is a function of V, such that $R = A V$, or,

$$P_s = \frac{w A V H}{550e} \quad \ldots\ldots (2)$$

The total head, H, is made up of the static head, the suction head, and the friction head. Equation (2) may then be written,

$$P_s = \frac{w A V (H_s + H_e H_f)}{550 e} \quad \ldots\ldots (3)$$

The friction head is an exponential function of the velocity and is also a function of the length of line through which the mixture is pumped. Therefore,

$$H_f = \frac{L f V^n}{d^p} \quad \ldots\ldots (4)$$

Substituting this value in Equation (3),

$$P_s = \frac{w A V \left(H_s + H_e + \frac{L f V^n}{d^p}\right)}{550e} \quad \ldots\ldots (5)$$

The output is also a function of the velocity, since it is proportionate to the rate, R, which is equal to $A\,V$. Therefore,

$$O = 133.3\,S\,A\,V \quad \ldots\ldots (6)$$

Substituting Equations (5) and (6) in Equation (1):

$$C = \frac{c_l}{133.3\,S\,A\,V} + \frac{c_p\,w\,(H_s + H_c)}{133.3\,S \times 550\,e} + \frac{c_p\,w\,(L\,f\,V^n)}{133.3\,S \times 550\,e\,d^p} \quad \ldots\ldots (7)$$

Differentiating Equation (7),

$$\frac{d\,c}{d\,V} = \frac{-\,c_l\,V^{-2}}{133.3\,S\,A} + \frac{n\,c_p\,w\,f\,L\,V^{n-1}}{550\,e \times 133.3\,S\,d^p} \quad \ldots\ldots (8)$$

in which, $A = 0.7854\,d^2$. This value is zero for a minimum, and,

$$V_e = \sqrt[n+1]{\frac{c_l}{c_p}\left(\frac{700\,e}{n\,w\,f\,L\,d^{2-p}}\right)} = \sqrt[n+1]{\frac{b\,c_l}{L\,c_p}} \quad \ldots\ldots (9)$$

Equation (9) can be written in a more convenient form, as follows:

$$\log V = \frac{\log b\,c_l - \log c_p}{n+1} - \frac{\log L}{n+1} \quad \ldots\ldots (10)$$

Therefore, V_e varies directly as the $(n+1)$ root of c_l and inversely as the $(n+1)$ root of c_p.

The practical use of Equation (10) is to determine the horse-power of the main pump driver that will produce at least the economic velocity for the existing costs of labor and power on any length of line which may be met in ordinary operation.

Economic Velocity Under Typical Service Conditions

In the Great Lakes region labor is highly organized and, consequently, is costly, and the power developed by Diesel engines is relatively cheap. Consequently, the economic velocity for dredging under those conditions is high, and it is important to supply the dredge with sufficient power to attain it.

In Table 2, Items 1, 2, and 3 give the economic velocity when the dredge is handling medium clay and the horse-power required for a 30-in. Diesel-electric dredge based on the labor and power costs which obtained in 1925 when the design was first considered.

TABLE 2.—Economic Velocities Under Various Conditions of Dredging.

Item No.	L, in feet.	V_e.	P_s.	P.
1	2 000	22.5	2 599	3 000
2	6 000	15.3	2 427	3 000
3	11 000	12.4	2 397	3 000
4	2 000	24.0	3 057	3 000
5	6 000	16.3	2 858	3 000
6	11 000	13.3	2 808	3 000
7	2 000	24.3	2 683	3 000
8	6 000	16.5	2 463	3 000
9	11 000	12.7	2 145	3 000

These values were determined by means of Equation (9) and (5), in which, $\frac{c_l}{c_p} = 3.940$; $b = 373$; $e = 0.70$; $n = 1.86$; $w = 68$; $f = 0.523$; $d = 2.5$; $p = 1.25$; $S = 0.067$; $H_s = 16$; and $H_c = 10$.

Anticipating an increase in labor cost, a 3 000-h.p. motor was installed. Under the conditions obtaining in 1930 the corresponding values of V and P_s are as given in Table 2, Items 4, 5, and 6. The actual velocity was checked at times in the field by injecting a salt solution at the dredge pump and observing the time required for it to travel through the pipe line, its passage at the discharge end of the shore pipe being detected by a galvanometer.

From the general Equation (10) developed for finding the economic velocity, it is possible to study the effects on the cost and shaft horse-power requirements, etc., for different values of the other factors involved. No effort has been here made to catalog partial unit costs of hydraulic dredging work as frequently printed, but rather to make a valuable contribution to the extension of the field of knowledge itself. A very careful and comprehensive estimate of the entire situation surrounding each piece of hydraulic dredging work is necessary in order to arrive at a worth-while unit cost. In the absence of such estimate, unit costs are generally misunderstood and misleading, because frequently, knowing the unit cost on one piece of work, a futile attempt is made to correct for the difference in conditions and the situation on another job by mere interpolation without fully recognizing the principles and fundamental conditions partly set out in this paper.

In developing Equation (10) only the costs of labor and power are included. To these must be added the maintenance and depreciation on the plant and the interest on the investment; the cost of shore labor for distributing the fill material; the cost of the attendant plant; wear of the shore discharge line; and the overhead. This cost of labor and power may vary from 30 to 70% of the total, depending on the nature of the material handled, local conditions of disposal, and weather hazards. It depends also on the cost of moving the equipment on and off the work, the cost of marine insurance, the cost for work done outside the section, and effects of shrinkage and of settlement. The miscellaneous items of cost are practically independent of the velocity and the power and may be considered either as current or fixed annual charges, depending on the annual output only.

Table 2 shows that the velocity and the horse-power required for economic operation is greater on short lines than on long lines, although the head is less. It is a characteristic of a centrifugal pump that the peripheral velocity of the impeller, and hence the revolutions per minute, is a function of the square root of the total head. For values of L from 2 000 to 11 000 ft. at the economic velocity of discharge, the head will increase in each case from about 130 ft. to about 220 ft. or, the square root of the head changes from about 11.4 to 14.8, a variation of about 23% using the upper value as a base. Table 2 shows that, in order to maintain the economic velocity the pump driver must deliver about 7% more power at this reduced speed on short lines than at rated full-load speed; on long lines, the pump must deliver 23% more power under the same conditions. In the case of a steam engine this can only be done by putting in an over-sized engine and using it at less than full load on long lines, or by using a smaller sized impeller, or by doing both. The disadvantage of this is that there is a lack of flexibility. This is also true in case of direct Diesel-engine drive. In either case the horse-power is propor-

tional directly to the revolutions per minute and the revolutions per minute cannot be varied independently of the horse-power without considerable loss. This condition can be met with an electric motor of proper design. Computations indicated that the value of this flexibility more than offset the loss of energy in the Diesel-electric conversion.

Effect of Decreasing the Unit Weight of Excavated Material

The weight, W, of dredged material may be determined from the formula, $w = S\ W + (1 - S)\ 62.5$, or,

$$S' = \frac{w - 62.5}{W - 62.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (11)$$

For example, if $w = 68$ lb. per cu. ft. and $S = 0.067$, by Equation (11) $W = 144.5$ lb. per cu. ft. This corresponds to a hard heavy clay. If $W = 124$ lb. per cu. ft., corresponding to a medium sand, then, by Equation (11), $S = 0.0894$.

In Equation (6), the output, $O = S\ A\ V = \left(\frac{w - 62.5}{W - 62.5}\right) A\ V$, varies directly as S, but inversely as W. Therefore, a decrease of 14% in the specific weight of the material corresponds to an increase of about 34% in the carrying capacity of the discharge, assuming that f remains constant. That is, for $W = 144.5$, $V = 22.5$, and $S = 0.067$, $O = 14\,700\ S = 14\,700 \times 0.067 = 985$ cu. yd. per hour, and for $W = 124$, $V = 22.5$, and $S = 0.0894$, $O = 14\,700 \times 0.0894 = 1\,315$ cu. yd. per hour.

Effect of Variation in f Upon V_e and P_s

Under typical service conditions for which the *New Jersey* was constructed, the value of f has been previously given as 0.523. If this value is changed to 0.420, the value of $b = \frac{700\ e}{n\ w\ f\ d^{0.75}}$, will change proportionately, and the economic velocities for Items 7, 8, and 9 of Table 1 are found by Equation (9).

These data show that for a decrease in f of 20%, when $L = 2\,000$ ft., V is increased 8% and P_s is increased 2%; and for a decrease in f of 20%, when $L = 11\,000$ ft., V is increased 2½% and P_s is decreased 12½ per cent.

Approximate Effect on V_e and O of Decrease or Increase in Diameter of Pipe Line

From Equation (9),

$$V_e^{n+1} = \frac{c_l}{c_p}\left(\frac{700\ e}{n\ w\ f\ L\ d^{(2-p)}}\right) = \frac{700\ D\ e}{n\ w\ f\ L} \div d^{(2-p)} \quad \ldots\ldots\ (12)$$

and,

$$V_e = \left(\frac{700\ D\ e}{n\ w\ f\ L}\right)^{\frac{1}{n+1}} \div d^{\left(\frac{2-p}{n+1}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ (13)$$

For typical service conditions, previously described, the exponent, $\left(\frac{2-p}{n+1}\right) = \left(\frac{2 - 1.25}{1 + 1.86}\right) = 0.262$; therefore, V_e varies approximately as $\frac{1}{d^{0.262}}$, and, for

$d = 2.5$, $L = 2$, and $V = 22.5$. If d is reduced from 2.5 to 2.0, then $V_e = \dfrac{22.5}{\left(\dfrac{2}{2.5}\right)^{0.262}} = 23.8$ for $d = 2.0$; that is, a 20% decrease in d corresponds to a 5.8% increase in V_e.

From Equation (6), $O = (133.3 \times 0.067)AV = 8.9AV$. For $d = 2.5$, $A = 4.90$, and $V = 22.5$. Consequently, $O = 8.9 \times 4.9 \times 22.5 = 985$ cu. yd. per hour; and for $d = 2.0$, $A = 3.14$, and $V = 23.8$, and, therefore, $O = 8.9 \times 3.14 \times 23.8 = 665$ cu. yd. per hour. This illustrates that, for a decrease of 20% in d, O is reduced 320 cu. yd. per hour, or 32½ per cent.

Effect on C of Decreasing the Diameter of Discharge Line

In Equation (9), $V = \left[\dfrac{b\,c_l}{L\,c_p}\right]^{\frac{1}{n+1}}$; and $b = \dfrac{700\,e}{n\,w\,f\,d^{(2-p)}} = \dfrac{B}{d^{0.75}}$. Therefore,

$$V_e = \left[\frac{c_l B}{c_p\,L\,d^{0.75}}\right]^{\frac{1}{2.86}} = c_l^{\frac{1}{2.86}}\,B^{\frac{1}{2.86}}\,c_p^{-\frac{1}{2.86}}\,L^{-\frac{1}{2.86}}\,d^{-\frac{0.75}{2.86}} \quad \ldots\ldots (14)$$

From Equation (7),

$$C = \frac{c_l}{133.3 \times 0.7854 \times 0.067\,d^2\,v} + \frac{c_p \times 68 \times 0.523\,L\,V^{1.86}}{133.3 \times 550 \times 0.7 \times 0.067\,d^{1.25}} + \frac{c_p \times 68 \times 26}{133.3 \times 550 \times 0.7 \times 0.067}$$

or,

$$C = \frac{c_l}{7.014\,d^2\,v} + \frac{0.01034\,c_p\,L\,V^{1.86}}{d^{1.25}} + 0.5142\,c_p \quad \ldots\ldots (15)$$

Substituting the value of V_e in Equation (14) for V in Equation (15), and simplifying:

$$C = \frac{c_l^{0.65}\,[c_p L]^{\frac{1}{2.86}}}{d^{1.737}}\left[\frac{1}{7.014\,B^{\frac{1}{2.86}}} + 0.01034\,B^{0.65}\right] + 0.5142\,c_p \quad \ldots\ldots (16)$$

The value of C for any economic velocity varies inversely as the 1.737 power of the diameter. The unit cost of labor, c_l, is also some function of the diameter; it is larger for large diameters. However, c_l may be considered constant for values of d between 2.0 and 2.5 ft. and a constant smaller value for diameters less than 2.0 ft. The unit cost, c_l, does not decrease proportionately with d, and even if it did, it appears in the numerator to a fractional power, while d appears in the denominator as a power greater than 1. Hence, the unit cost may be decreased by increasing the diameter of the discharge of the dredging pump.

Effect of Operating at Velocities Higher and Lower Than V_e

The value of V for minimum cost is given by Equation (9) assuming $n = 1.86$; thus,

$$V_e^{2.86} = \frac{c_l\,b}{c_p L} \quad \ldots\ldots (17)$$

Under the typical service conditions previously described, the shaft horsepower, P_s, is determined from the relation:

$$P_s = 22.54\ V + 0.1442\ V^{2.86}\ L \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

The value of P_s corresponding to a minimum value of C occurs when $V = \sqrt[2.86]{\frac{c_l\ b}{c_p\ L}}$. Substituting this latter quantity in Equation (18) and writing D for $\frac{c_l}{c_p}$,

$$P_s = 22.54\ \sqrt[2.86]{\frac{D\ b}{L}} + 0.1442\ D\ b \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

The economical shaft horse-power varies directly with D and b and inversely with L. For all values of V greater than, or less than, $\sqrt[2.86]{\frac{D\ b}{L}}$, C will increase. Since P_s varies directly with V, the application of any shaft horse-power greater than that corresponding to $V = \sqrt[2.86]{\frac{D\ b}{L}}$ will increase the unit cost instead of reducing it. Fig. 1 shows the effect on C of a change in V and P_s for a particular instance, with L equal to 6 000 ft. It illustrates most clearly how definite Points V_e and P_{se} are, and how sharply the value of C rises with the application of power either less or greater than P_{se}, the economic power.

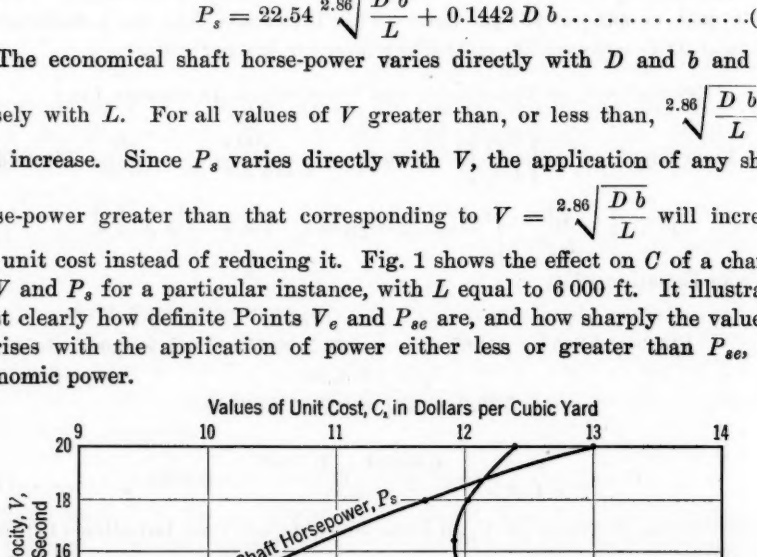

FIG. 1.—CURVES SHOWING THE RELATIVE EFFECTS OF VARYING THE VELOCITY OF DISCHARGE AND THE SHAFT HORSE-POWER.

EFFECT OF BOULDERS IN HYDRAULIC DREDGING

The unpleasant consideration of this element of the dredgeman's work was left to the last so that the foregoing engineering principles and finer analysis might be given calm and deliberate thought. Boulders are frequently mixed with clay and sand in natural deposits. Many times they are promiscuously distributed; sometimes they are more or less uniformly scattered; and at other times they are found in nests as if to make a concentrated attack on the dredge for disturbing their peaceful seclusion of millions of years. A relatively small percentage of boulders by volume in the proposed cut eliminates it for the hydraulic dredging method of excavation. In fact, the only sufficient reason

for considering the hydraulic dredging of ground containing boulders is to secure that part of the work not involving the handling of them.

All boulders are highly objectionable in this method of excavation, but the size most detrimental is that critical one which will pass through the opening in the cutter and will not pass through the opening in the pump flyer. It becomes lodged and sometimes tightly jammed, thus badly unbalancing the flyer and making it necessary to shut down the dredge for the removal of the boulder. These are expensive delays and, in terms of unit of excavation, may cost as much as $200 to $500 per cu. yd. for the boulders encountered. Boulders that are too large to enter the cutter, or become jammed in it, or smaller boulders that will pass through the pump and discharge line, are not so objectionable; but they are highly undesirable, and their presence seriously upsets all the calculated economies set forth in this paper.

There is, of course, a relation between the size of opening in the cutter and that in the pump flyer. The cutter designed for maximum efficiency in clay will have a maximum opening or clearance between blades of approximately six-tenths the diameter of the pipe line. For low-speed dredge pumps the minimum opening or clearance through the flyer vanes would be approximately five-tenths the diameter of the discharge pipe, and for high-speed pumps somewhat less, because of the flatter or less radial shape of the vanes. As the efficiency of the main dredge pump is so important, an efficient design of the impeller cannot well be compromised for this boulder condition. Similarly, the efficient design of the cutter is very important and, therefore, these clearance dimensions for both cutter and pump are more or less fixed. The particular size of boulder that will become lodged in the cutter or in the pump is the critical size and, of course, this varies with the size of the dredge.

The frequency of delays to the dredge caused by the removal of boulders is, of course, in proportion to the percentage of the critical size of boulders in the area to be excavated, and also to the output of the dredge. While the presence of boulders has a great effect on the cost of the work, there is no formula, either theoretical or empirical, that is applicable for determining the effect. Only by careful analysis and long experience can this element of the cost be even roughly estimated for any job.

The critical size of boulder for the *New Jersey* is 3 to 5 cu. ft., or 400 to 700 lb.; but, at times, and on short-line work, much larger ones are taken from the pump. For example, a granite boulder weighing 1 050 lb. was taken from this pump at Toledo, Ohio, on July 5, 1929, while dredging heavy hard clay mixed with sharp gravel. This combination of material is somewhat unusual and difficult to handle. Mysteriously, the minimum dimension of this large boulder (19 in.) was more than an inch greater than the maximum distance between the cutter blades, and if this is accounted for by springing these heavy short blades, it furnishes a new measure of the forces to be resisted and the sturdy design required.

The Design Elements

Having determined, from the preceding analysis, the relation between the different factors involved and their relative importance for economic operation,

the conclusions, tempered with experience, have been applied in the design and construction of the *New Jersey*. Another factor considered of great importance, of course, is the general time efficiency of the entire dredge. Such dredges are usually operated 24 hours per day, and the time efficiency is the ratio of the number of working hours to the 24 hours, and for obvious reasons too numerous to mention, this has been kept in mind in the design of every unit of the dredge.

Having determined that a dredge with a 30-in. discharge, with Diesel-electric drive, is the most economical within practical limits and the best commercial type of the available sizes or types of power installations, the problem remains to design the component parts necessary for the successful operation of the entire installation.

Hull.—The first consideration in design, of course, is the hull, which must be constructed so as to provide buoyancy for the weight of machinery, fuel, supplies, etc. It was necessary that it be sufficiently rigid to withstand the vibrations set up by the Diesel engines, as well as the stresses incident to dredging operations, and that it be able to support the dredge without failure if stranded on an uneven bottom.

The limitations were that the hull should not draw too much water which would eliminate it from working on shallow projects, and that its length and breadth should not exceed the capacity of locks in canals between the Great Lakes and the Atlantic Coast. It was found impossible to keep within the required beam, so the hull was constructed with a detachable side section to bring it within the required width. The dimensions finally decided on were: Length, 245 ft.; beam, 50 ft.; depth (moulded), 14 ft. 4 in.; and draft (working condition), 9 ft. 3 in.

The hull proper is divided into compartments by a system of water-tight bulkheads. This affords maximum protection against sinking from collision or other mishap as well as protection against damage to the electrical equipment should any part of the hull be flooded. The side compartments are used for the storage of oil, water, and supplies; the larger center compartments are used for the machinery. The main pump and main pump motor are in separate compartments to avoid damage to the motor in case the pump should be broken by a boulder, or from some other cause. These bulkheads add considerable strength and rigidity to the hull structure.

In addition, there are two longitudinal trusses, at 23-ft. centers, the full length of the hull. They are extended at the bow to form the gantry for supporting the dredging ladder, and at the rear they support the stern spud anchors. The depth of these trusses is less than that ordinarily found in bridge practice. It was desired to keep down the height of structures above water to reduce wind resistance and maintain low bridge clearance. To have given the truss greater depth would have necessitated running it through the roof of the lower house which would have made it unsightly and left opportunity for leaks. In addition, it would have projected into the living quarters which would have made that part of the layout quite complicated. The additional weight due to the reduced depth is not great and adds to the rigidity of the entire structure.

Another novel feature is the shaping of the ends of the hull. This is done to reduce towing resistance. As early as 1914 the writer's Company had tests made at Rensselaer Polytechnic Institute, Troy, N. Y., to determine the relative resistance of square hulls with ends varying in shape. The models had square ends with differently shaped rakes. It was found at that time that a section of compound curvature increasing at the bottom had the least resistance. Later tests made by W. C. Sadler, M. Am. Soc. C. E., at the University of Michigan, developed the fact that a rounding of the deck section still further reduced the resistance. The results of both these series of tests have been included in this design.

Another feature of the hull design is the recess placed at the after starboard corner to accommodate the stern ball connection in the discharge line. Two methods are ordinarily used for this connection, either (1) to bring the discharge pipe through the hull near the water-line and support the connection by an **A**-frame, or gantry; or (2) to bring the pipe above deck and use a swinging elbow in a gooseneck. Both methods have serious faults. The first has the disadvantage of a connection that is more or less inaccessible due to its being in or near the water; the second involves short-radius elbows, and brings the discharge too high above the water. The new method meets the objections to both, eliminating the elbows and putting the discharge pipe at the proper level for joining to the pontoon line.

Deck House and Crew Quarters.—The main deck house is of steel and provides a complete enclosure for the machinery, leaving a deck space of approximately 9 ft. on each side. This space has guarded bulwarks 30 in. high around the ends and extended about 6 ft. along each side, with chains supported by removable steel stanchions between the bulwarks. The roof is supported by the longitudinal trusses. The doors in the lower house are of steel, and the windows are fitted with wire glass in steel sash.

The upper house rests on the steel roof of the lower house. It is of wood construction throughout and is divided into 39 rooms, comprising 18 staterooms, 3 toilets, 3 baths, 2 washrooms, 2 dining-rooms, 2 storerooms, wireless room, office, galley, pantry, room for ice machine, refrigerator room, officers' lounge, crews' "social hall", and laundry.

It has been found that comfortable quarters for the crew on a machine of this kind where the men spend most of their time on board, is a great asset; it keeps them satisfied and more inclined to stay with their job. The provision of an officers' lounge and a social hall for the crew may appear extravagant at first, but it must be realized that it is only necessary to take a very little from each of the staterooms to make this space available, and that the common social halls afford opportunities for recreation which could not be made possible in individual quarters.

All the rooms are heated, lighted, and well ventilated, and open into a common corridor in the middle of the house. Over this corridor for its full length is a monitor with ventilators. The galley is fitted with sink, tables, lockers, shelves, and an oil-burning range. Complete culinary equipment is provided for a crew of fifty men.

Cutter and Ladder.—A heavy plate-girder ladder of sufficient length for a digging depth of 50 ft. is hinged to brackets in the bow well, with its outer end suspended through wire-rope tackle blocks from the bow gantry extension of the main trusses. The lower end of the ladder is formed by the steel casting suction head made in two pieces with two bearings for the cutter shaft about 10 ft. apart. The outer bearing extends well inside the cutter head and consists of a sealed and water-lubricated cutless bearing. The suction head has a pressure-water chamber with jets directed into the suction inlet to assist in freeing a clogged pipe. Lugs are provided for the purpose of anchoring the ladder suspension tackle and the swinging sheaves. Heavy flanges serve to tie the suction head thoroughly to the structural part of the ladder. The upper ladder end has three heavy annealed steel castings which form the ladder hinges and foundations for the cutter motor and gear box.

The cutter head (Fig. 2) was designed and built by the writer's Company. It is an integral alloy steel casting with eight spiral arms connecting the hub and back ring. Replaceable chrome vanadium blades are riveted to the arms. The hub is taper-bored and keyed to a 13½-in. cutter shaft which is supported by two bearings in the suction head, two in the gear box, and four intermediate bearings mounted on transverse ladder girders. The lower bearing is designed to transmit the axial thrust of the shaft to the ladder.

The 600-h.p. cutter-drive motor is mounted on the ladder and drives the cutter shaft through a flexible coupling in combination with a slipping friction. The first set of reduction gears is of the herring-bone type; the second set, consisting of the cutter shaft gear and its pinion, is of the single helical type. Both sets are encased and run in oil, with all gears and pinions mounted between two bearings built into the gear casing. The cutter shaft is made in six sections held together with bolted and keyed couplings.

Suction and Discharge Pipe.—The suction pipe has an inside diameter of 34 in., and enters the hull through a packed and water-sealed ball joint which is bracketed to the end of the well plating and the deck concentric with the ladder trunnions. A readily removable manhole section next to the pump has a hydraulically-operated hinged cover, the mount of which permits it to function as a suction line relief valve.

The discharge pipe has an inside diameter of 30 in., and is located immediately outside the house on the starboard deck. It is connected to the discharge line with a rubber-packed, quick-opening ball joint mounted in a gimbal ring, the deck being tapered off to the stern to receive the bracket and ring. The joint is provided with compression spring bumpers which contact with a structural frame to limit its transverse angular movement. A gate-valve section is placed in the line immediately forward of the ball joint, and there are two manhole sections adjacent to the pump discharge. Replaceable plate liners are spot-welded in the lower half of each straight pipe section. All cast-pipe fittings on the dredge have cast-alloy steel liners. The discharge pipe sections are held together by special rubber-sealed pressure joints.

Pump.—The pump (Fig. 3) is somewhat unusual; a number of novel features have been introduced in its design. It is of the square-casing lined type recommended by J. H. Polhemus, M. Am. Soc. C. E., who has had a

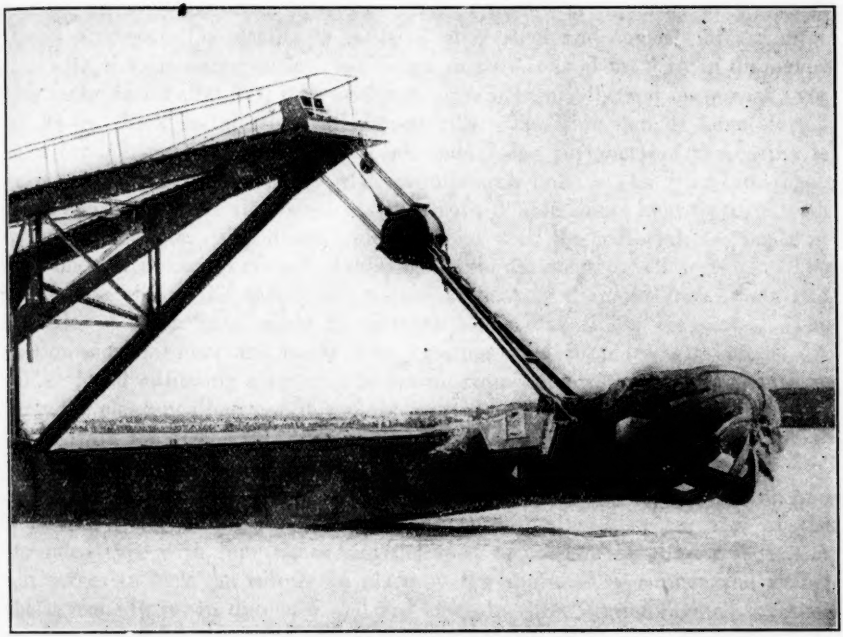

FIG. 2.—VIEW OF CUTTER HEAD.

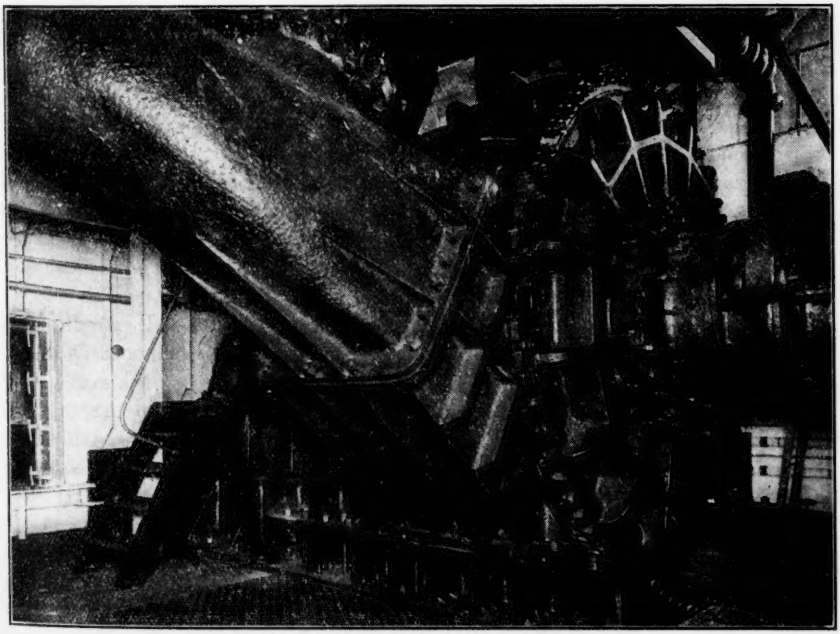

FIG. 3.—THIRTY-INCH DREDGE PUMP.

marked degree of success with that type of pump on the Port of Portland (Ore.) dredges. The casing is made of alloy steel and consists of an outer ring with the discharge outlet cast in one piece. It is about 9 ft. in diameter. The inner face being flat, it is stiffened circumferentially and cross-wise over the 3½-in. thick metal with rail-shaped ribs. The side flanges have slotted holes for bolting to the front and back ends. The periphery of the casing is lined with plate sections held with countersunk bolts. The front and back heads are well-ribbed alloy-steel castings with double liners held to them with countersunk bolts. The liners which contact with the material, are made in a number of pieces to permit obtaining maximum wear on all sections. The liners adjacent to the casing are somewhat heavier than the disk liners and are alloy-steel castings made in sections. The disk liners are plates. The suction-mouth liners are heavy iron castings held in place with studs. A rubber-faced adjusting ring may be set up from outside the suction mouth to limit the clearance between it and the runner. This tends to prevent a circulation from the discharge pressure chamber to the suction inlet, thereby maintaining the pump efficiency and eliminating wear.

For experimental purposes, there are two runners; one with a square, and the other with a long-sweep, throat. Both are alloy-steel castings of the shrouded type with four vanes and threaded to receive the runner shaft. A nut serves to lock the runner in place on the shaft. The runner and thrust shafts are 14⅜ in. in diameter and are made in three sections bolted together through flanges forged with the shafts. The intermediate or short section is removable to permit of readily withdrawing the runner shaft through the back head. The shaft has a replaceable sleeve where it passes through the water-sealed stuffing-box in the back head. A motor-operated turning device consisting of a worm and wheel which may be readily thrown in or out of engagement, serves to assist in backing the runner off the shaft or to lock the shaft against turning. The thrust shaft has one collar which transmits the thrust to the base through an over-sized two-way, oil-lubricated, water-cooled thrust bearing in combination with a radial bearing. The radial bearing next to the pump back head is of the same type.

Three heavy cast-iron bases are securely bolted together to form a rigid foundation for the motor, pump casing, and bearings.

Bow Winch.—The bow winch consists of five drums loosely mounted on a single shaft. The middle drum hoists the ladder and is driven from a 110-h. p. motor with control for dynamic lowering. The two drums next to the ladder hoist drum are used for swinging the dredge in normal operation, and the two outer drums may be used for side-breasting or for crane service. A 125-h. p. motor drives the swinging and the auxiliary drums. The motor pinions are mounted on stub shafts. Each set of first-reduction gears are of the herring-bone type encased and running in oil. The second set of intermediate gears and the pinions which mesh with the drum gears, together with an independent friction and brake, for each drum, are mounted on a single intermediate shaft which has a winch head on each side of the house. The intermediate shaft is made in coupled sections for alignment purposes. The hoisting drum is grooved for its rope; all other drums have plain barrels. All spur gears have cut teeth.

The frictions, which are of the internal expanding, reversible driven type, are air-operated. The machinery with the driving motors is mounted as a unit on a substantial, structural steel base.

Stern Winch.—The stern winch is mounted on the deck aft of the main engine hatch and forward of the spuds. It consists of two drum shafts upon each of which is loosely mounted a grooved spud drum and a plain barrel auxiliary drum. Each drum has an independent friction and brake which, together with the drum gear pinions and intermediate gears, are mounted on an intermediate shaft, coupled in three sections, with a winch head on the port side. The first reduction gears are of the herring-bone type encased and running in oil. The auxiliary drums may be used with the forward auxiliary drums for breasting the dredge, for warping the pontoon line, or for auxiliary crane service. The gears, frictions, and other winch details are similar to those of the forward winch. The machinery and motor are mounted as a self-contained unit on a substantial, structural base.

All clutches and brakes for the bow and stern winches are controlled from the pilot-house which is located above the crews' quarters at the forward end. Each lever line is distinctly marked. If desired, the stern winch (including the motor) may be controlled from the stern as well as from the pilot-house. The pilot-house is the full width of the deck-house and has windows on all sides. It houses the controls for the cutter, ladder hoist, forward and stern winch motors, a push-button cut-out for the main pump motor, as well as all meters and gauges for the motor and main pump. The pilot-house is in communication with the engine-room and the other parts of the dredge through signal, telephone, and speaking-tube systems.

Spuds.—There are two spuds mounted in lined steel casting keepers in wells cut into the stern with a space of approximately 20 ft. between the upper and lower keepers. The upper keeper reactions are taken directly into the hull trusses. The stern of each keeper has a pin-connected gate which may be opened to unship the spuds. The spuds are 48 in. square, made with corner angles and side-plates, and two internal latticed trusses in the fore and aft direction. The lower spud end is of novel design; it consists of a heavy cylindrical steel casting, the outer point of which is clamped to, and rotates about, an inner point to which the square structural spud is securely riveted. Bushings and washers which form the contacts between the inner and outer points, are made of special brass. The spud-hoisting sheave is mounted in the upper end of the inner casting and is thoroughly rope-guarded. The rotating-point feature relieves the spud of all torsional stresses which otherwise would have to be transmitted through the spud to the keepers when the dredge swings.

Another very unusual feature is the provision for pinning up on the spud as is done on the forward spuds of a dipper dredge. This assures a firm grip for the spud in any reasonable bottom. The point remains fixed while the spud rotates, thus tending to prevent loosening the grip by enlarging the hole which has been made by the spud point.

The design of the spud machinery is such as to utilize the weight of the idle spud for "pin-up" reactions on the digging spud. The "pin-up" sheave is mounted in a steel casting bracket riveted to the upper spud end.

Power Plant.—All main dredging units are driven from a 600-volt direct-current power line energized by four 800-kw. parallel-operated, direct-current generators. Each of the latter is driven by an 1 150 b. h. p. vertical, 6-cylinder, 180 rev. per min., Diesel engine of the 2-cycle, air-injection, cross-hand type. The engines are mounted on heavy longitudinal girders which are tied into the deep transverse floor system with plate intercostals. The generators and outboard bearings are mounted on steel-casting bed-plates.

The main Diesel engines are right and left hand with the operating gear for each on the inside. The air compressor, scavenging pumps, lubricating oil pumps, and fuel-oil injection pumps are built integral with the engines. A fresh-water cooling system consisting of re-coolers, filters, sea and fresh-water pumps, and a tank into which the water is discharged from the engines and redrawn by the circulating pumps, maintains the engines at a proper temperature by circulating water through the cylinder jackets and pistons. If desired, sea water may be used for cooling purposes in which case a small pump preserves a constant level in the discharge tank by discharging the cooling water overboard.

The dredge is equipped with 53 motors having a total normal rating of about 4 600 h. p. The main motors are operated from the 600-volt main generator busses, and this includes all motors of 5 h. p., or more, except the 7½-h. p. bilge-pump motors. The other motors are for auxiliary service and are of units less than 5 h. p. They are operated from the 125-volt bus which is energized from a 75-h. p. motor generator set, or the 75-h. p. Diesel engine set.

The main pump motor is of the adjustable speed type and will normally develop 3 000 h. p. continuously. Resistance is used for starting duty only, which means a sufficient period of time to fill the pipe line. Push-buttons in the engine-room serve to regulate the speed of the main motor through a motor-operated field rheostat between the limits of 250 and 360 rev. per min. This range is sufficient to allow for all reasonable service variations due to the pumping of different grades of material or variable line lengths with the same runner. This type of motor and control permits of efficient operation at full-load rating at any speed within the range, thus using the entire output of the power plant for variable duty, which assures maximum output at all times. The motor may be stopped by a push-button in the operating room.

The cutter motor has a full-load continuous rating of 600 h. p., and will develop overloads for time intervals up to a maximum of 1 100 h. p., for 5 min. This is considerably more power than has ever been applied to a cutter and the method of control is entirely new. The motor is of the adjustable-speed, double-armature type with resistance for only heavy-duty starting. The normal speed range with armatures in parallel is from 250 to 500 rev. per min. for full loads or overloads. With the armatures in series, the speed range is from 125 to 250 rev. per min. with about 50% of the normal load or overload ratings. This arrangement with the 4:1 speed range gives efficient and flexible control. The power developed by the motor at slow speeds is ample to satisfy the service condition, as the slow cutter speeds are required for easy digging only. The cutter and swinging motor controls are interlocked, which automatically slows

down the swinging speed when the cutter is overloaded and restores it after the cutter motor has pulled through the overload condition.

The swinging motor has a continuous rating of 125 h. p., and is operated from the variable voltage motor-generator set with a Ward-Leonard system of control for starting, stopping, and regulating the speed from zero to 700 rev. per min. This system is used on the swing motor because of the necessity for a very wide speed variation and more reliable protection for the cutter motor through the interlocking control than could be obtained by a resistance control. A solenoid brake is provided on the swinging motor to prevent the dredge from drifting through wind or current action.

The ladder and spud-hoist motors are of the series-wound type with magnetic control and dynamic lowering. Solenoid brakes are provided, which set on the neutral position of the control lever. Both motors are controlled through the pilot-house, but the spud-hoist motor may be controlled from the stern if unlocked through the pilot-house controller. Both motors have a 75°, half-hour, 110-h. p. rating.

Miscellaneous Equipment.—There are two 10-ton, hand-operated traveling cranes, of 21 ft. span, mounted on overhead runways and supported by the truss posts. The runways extend over the upper end of the ladder to the bow gantry. For making heavy lifts, both cranes may be used together at any point in the length of the runway. There are six 5-ton, deck-mounted, cargo cranes, three forward and three aft. Blocks, lead sheaves, and anchorages are provided for operating these cranes by hand, from the winch head, the auxiliary drums of the bow, or the stern winches. A jib crane is pin-connected to the suction head at the lower end of the digging ladder, and is designed to fold back so as to be held by the ladder suspension bars when not in use.

A 3-ton, stockless anchor with 150 fathoms of stud-link chain, operated from a 30-h. p., worm-driven anchor hoist, with a locking gear and hand-operated brake, is located between the spuds on the deck, with a chain locker below the deck. A 30-h. p. capstan, with hand-operating mechanism, is located on the deck at the stern.

Application of the Gyro-Compass to Dredging Work

Under numerous circumstances, especially when working in the lake at some distance from shore and at times when weather conditions prevent the use of ordinary ranges placed on shore, it is desirable and often essential that a means be provided on the dredge both for determining its location with reference to fixed points on shore and the width and direction of the cut being made by the dredge.

A partial solution of this problem was suggested in the use of the compass. To be of value in this respect the compass would have to be under the constant view of the operator and, because of the many cables carrying heavy electric currents which pass through and about the operating room, the magnetic compass proved to be of little value as it could not be adjusted to maintain true readings.

The next suggestion was the use of a gyro-compass which is not affected by the presence of electric currents and which has an additional value in that its

readings are always given in true azimuth. In this manner the compass readings can be adapted to a survey based on co-ordinates the axes of which are parallel to and perpendicular to the true azimuth without correction.

The installation consists of a master compass which contains the gyroscope and all its mechanism, together with one or more repeaters, which has the advantage of providing a relatively small instrument in the repeater which can be placed in a convenient location. These repeaters are provided either with fixed horizontal compass cards or are mounted in gimbal rings for tilting in the plane most convenient for reading. An azimuth circle attachment is also provided by use of which the position of the dredge may be determined by triangulation, with fixed points on shore, the co-ordinates of which are known. More exactly, this process gives the location of the compass from which, by reading the heading of the dredge, the exact location can be determined by means of a small template of the same scale as the survey map.

The installation of this instrument proved a complete solution of this problem, and it was found that the dredge could be located within reasonably accurate limits by this means. Likewise, having established the location of the dredge, the cut can be maintained in any predetermined direction and of any predetermined width by controlling the swing of the dredge an equal distance either side from a predetermined average heading or direction of the cut.

After this installation was made and found to be satisfactory, a further addition was made to the installation which consisted of a recording repeater. The record of this repeating instrument was made on a paper strip, 7 in. wide, which gave the azimuth readings across the sheet, passed under the recording pen at a rate of about $\frac{1}{4}$ in. for each 10 min. of time. The clock mechanism was operated by an 8-day movement, and the record roll was of sufficient length to last a month.

In ordinary use on board ship this record is almost a straight line, showing only variations from the established course of the vessel due to windage, wave action, or other disturbing forces which alternate with the corrections applied to these deviations either by the wheelsman or by the automatic steering mechanism which is often applied to it. The curve produced by this recording and dredging operation, however, is quite different. It consists of a series of diagonal lines which record each swing of the dredge, indicating the azimuth reading at the end of each swing. Delays are recorded by a straight line in the direction of the chart so that a complete record of the operation of the dredge is maintained, indicating the exact number of minutes occupied in actual dredging, during which time the dredge is swung either in one direction or the other, and the actual delay time.

In addition, the operation of moving the dredge ahead in the cut by walking it on the two stern spuds leaves a characteristic record. The swing of the dredge is interrupted on the chart by a delay of about 1 min. or 2 min., during which time one spud is lowered and the other raised. This is followed by a continuation of the swing line through a certain number of degrees, and followed again by a delay of from 1 min. to 2 min., during which time the spuds are reversed. From the distance between the centers of the spuds and the angle

moved through in the walking operation, the number of feet which the dredge will be advanced, can be determined. Checking the computed advance of the dredge over a considerable period of time by location from fixed points on shore, it was found that this method was especially accurate. This compass record gives an accurate determination of the width of the cut and the advance of the dredge so that, knowing the elevation of the bottom before and after dredging from soundings, the quantity of material moved can be checked with fair accuracy from day to day and from hour to hour.

As an essential part of the dredge operation, especially designed pontoons lashed together support the discharge pipe, and a special derrick boat at the discharge end permits of a wide range of free flow from the end of the pipe when discharging over a breakwater, or otherwise.

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HIGHWAY LOCATION: GENERAL CONSIDERATIONS*

By Gibb Gilchrist,† M. Am. Soc. C. E.

Synopsis

Highway location problems are treated very broadly in this paper. Basically, the subject has been divided into two parts. The first part, dealing with physical aspects, emphasizes the dangers attendant upon locating new highways so that they follow old routes. The advantages of aerial photography are mentioned, and aerial survey methods adopted by the Texas State Highway Department are described briefly.

In the second part, the paper describes the non-physical aspects of highway location, such as safety, economy, and utility of operation. This section is concerned with the influences of local and through traffic demands on the location problem. The paper also contains a brief comment on financing.

Introduction

When the highway location problem is solved, the highway problem itself is well on its way to solution. Early in 1929 one of the influential and best informed members of the Texas Legislature sent a questionnaire to the State Highway Department, requesting information on various phases of its program. One of the questions was as follows: "Give the average life of roads built of the various approved types of construction, such as reinforced concrete, concrete with an asphalt surface, macadam, gravel, etc." The answer given was that the life of any particular type of road depended, first, on its location and, second, on the width of its right of way, and that the type of surface was of secondary consideration. Of course, in this reply the writer had in mind the utility of a highway as applied to its use as an important traffic-way. He had no reference to the stability of any particular type of surfacing, but wished to bring forcibly to the attention of those engaged in enacting highway legislation that the life of a highway should be considered as the length of

* Presented at the Joint Meeting of the Highway and Construction Divisions, Dallas, Tex., April 25, 1929. Written discussion on this paper will be closed in March, 1931, *Proceedings*.

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time during which it would or could be used as an artery of traffic. He wanted, furthermore, to emphasize the fact that necessity and public opinion would demand that such a highway, either now or in the future, be located with a fair degree of economy.

Physical Aspect of Highway Location

In laying out a State highway the engineer has been too prone to follow what has been generally and popularly termed the "line of least resistance." For example, he has followed the right of way of a railroad or interurban, or if there has been no such easy route, he has chosen to locate one along section lines or property lines without any particular regard for the terrain which he traverses or for the suitability of such location for possible future heavy traffic. This policy of the engineer may be partly justified by the explanation that a majority of the locations for highways has not been made by engineers. Too often they have been determined at mass meetings or conferences of citizens who were thoroughly eager and sincere in their determination of routings but who failed to calculate, or to have the economy of either road location or construction determined by experts. This is not stated as criticism because without such action by those interested in road improvement in the various States, the Nation would be much less advanced than it is to-day. Furthermore, the writer knows of very few cases in which the location or construction of any type of road or highway has not at least paid for its investment over the period in which it has been used. How much greater return, however, could have been obtained from the use of highways if, as time went on, the cost of traveling a much greater distance could have been saved; and with how much more safety the public could have reached its goal!

Almost every important railroad is rather closely paralleled by a State highway. This, of course, is largely due to the fact that the railroads have developed the population centers and the demand for highways has been along the same general line. However, with the location of State highways parallel to the railroads, the engineers have been too ready to accept the railroad location as being along the best ground. This is usually true for a steam road, but problems of highway location are considerably different from those confronting the railroad locating engineer, and much distance can be saved to highway travel in country where railroads have considerable trouble making 1% and 1.25% grades and have to make distance for gradient.

The writer was once in charge of a railroad location in which the limiting grade was 1.25 per cent. The final located route between two points measured 14.5 miles, and more than 75 miles of preliminary lines were run and much adjustment was necessary to obtain that. Between these same two points a State highway is now being planned, with a maximum 3% or 3.5% grade, and the total distance will be shortened to 11.5 miles. Instances of this kind are very common in the State of Texas.

Aerial Photography.—Any discussion of highway location by one familiar with the situation in Texas would be incomplete without a reference to the use of aerial photography. In reconnaissance the Texas Highway Department is using such service to a great extent and has found it very valuable. No

other improvement in the field of reconnaissance or location has contributed so largely to the efficiency of engineering.

The usefulness of an aerial survey begins at the first inception of the work and continues until the last shovelful of dirt is in place. It puts in the hands of the engineer many of the details that are to be found on the ground and in addition it gives him a broad view of the field of operation that has never been available in any other way. He may make locations directly on aerial mosaics; he may study minutely the locations of drainage structures and other proposed improvements; he may calculate stream flow and plat drainage areas without the use of field operations; and throughout the progress of his work the aerial survey stands by as a constant check upon the more exact field calculations.

The Texas State Highway Department has secured some very excellent results by making a location on the ground after having laid it out on an aerial map. The aerial photographer furnishes an approximate scale of each map, and in most cases these determinations are found to be very accurate. The aerial maps are especially useful in crossing streams and determining routes through cities and towns. Where little information is available from previous surveys or maps it is always economical to have an aerial map made. Such maps on an approximate scale of 700 ft. to an inch cover a strip of terrain about 6 000 ft. wide for single-flight, or 9 000 to 10 000 ft. wide for double-flight, maps. The cost is from $14.00 to $17.50 per lin. mile for a single-flight and approximately twice these amounts for double-flight maps.

After the completion of reconnaissance either by aerial photography or by preliminary lines, the best plan is to locate the final line on the ground, making such shifts of tangent or curve as may better fit the terrain from the actual field measurements. Very little difficulty has been experienced in laying out locations from aerial reconnaissance after walking over the proposed line once or twice, noting any physical conditions which might affect the location and then taking them into account. After the line has been laid out on the ground to complete what might be termed the locating problem, the engineer has only to prepare his alignment and right-of-way map, showing thereon such land as may be necessary to enable field notes to be prepared and giving the name and abstract reference to each tract of land. These plats are usually made on a 400-ft. scale which has been found to be the most adaptable to the use to which the map will be put.

In general, with regard to the location of a State highway, it is of prime importance to select the most direct line between two objectives. The line should be so located as to provide a minimum of waterways and the easiest grades and curves possible for the terrain. An important consideration is to select a line that will give not less than a 500-ft. sight distance between the eye of the driver and the top of an approaching automobile. Easy curves are not objectionable in order to increase sight distance where they do not materially lengthen the route. Such locations should be made and grades prepared with the primary object in mind so that, regardless of the development of the country through which the highway might pass, any surfacing placed on

the prepared grade would be only a stage of improvement, the ultimate life of which would depend upon traffic requirements of the highway. This was what the speaker had in mind in answering the question by the Texas State legislator. In other words, a highway properly located with a grade line of sufficient flatness to meet the exacting demands of increased traffic, and with curves in easy stages, will have its ultimate life more permanently assured as years go by and its condition should be better at the end of fifty years than at the end of five. With these considerations in mind the locating engineer, in considering his route between objectives, should disregard "the line of least resistance" unless he is prepared to see the day come within his own lifetime—or better still, within a decade—when the route he lays out will be relegated to the status of a lateral or community road, or be entirely abandoned. When such time comes, very likely it will be forgotten that the original location was perhaps determined by considerations purely local, and the necessity for relocation or abandonment, may, without much fear of contradiction, be explained as the correction of a mistake in engineering.

The problem of locating a highway with respect to topography, drainage, grade line, sight distance, and all those phases which must be studied in their entirety, is one that must command the expert attention of the locating engineer and of the construction engineer. Nevertheless, its difficulty of solution is secondary to that of the economy of location with regard to traffic, the distance that must be traveled, and safety.

Location with Respect to Safety, Economy, and Utility of Operation

The writer believes in the democratic principle that the voice of the majority should rule. He has never known it to fail, that where the people of any State or Nation were acquainted with all the facts with regard to any problem, they did not eventually (by their majority) vote to solve this problem in a way that the thinking people of such a community or commonwealth might determine that it should be solved. The people should have what they want, even if it costs more, that is, when they have decided with all facts before them.

The building of State highways is a public enterprise and the solution of the problems with regard to such a public enterprise should be entered into with the idea that it would be inadequate if it did not meet and satisfy the demands of the thinking public and if it were not in line with sound public policies.

Highway engineers are very often concerned with the problem of routes leading into cities and towns. This problem has been uppermost in the minds of the people for some time and public sentiment is now undergoing a rapid transformation in regard to highway location in population centers, large and small. In the State of Texas much prominence has been given to certain isolated cases of location where the State Highway Department has, for safety purposes, undertaken to re-route certain highways. Charges have been made that the officials and engineers of the State Department have in mind the laying out of a highway system that would isolate the cities and towns. In order to combat this erroneous impression the Highway Department has gath-

ered some statistics and has made some calculations. The State now (1929) has a highway system of approximately 20 000 miles as measured along present routes. When this system has been economically located or relocated the mileage will be about 19 000 miles and not only will all the control points be reached, but this system will reach directly, without material loss of distance, 97% of all towns, the population of which exceeds 200. Included in this percentage are all towns that have a prospective main-line highway passing through their corporate limits or, in case of no corporation, within four blocks of the main business street.

Through Traffic.—The general argument for the route of a State highway to pass through the business district of a town or along its most important street is that business is brought to the town by reason of the traffic on that highway. There is no doubt that certain classes of business establishments do profit to a certain degree by transient trade, but it would seem that a town depending to any degree on such trade would lack the foundation necessary for stable and continuous growth; that is, an established and a certain rural trade territory. The highway or road problem for any community will not be solved until every village and town is either provided with a highway or with a durable connection to the highway. The farmer or community trader will then have access at all times, not only to the State highway but to the main business center of his trading point, and the residents of the community will have a means of leaving and entering. It seems contrary to sound economic judgment and good public policy to deflect the location of an important State highway, carrying 2 000, or perhaps 5 000, vehicles per day, 2 or 3 miles out of the general course in order to pass through the main business district of a community which would perhaps not originate more than 100 to 200 vehicles per day. The problem should require and demand the careful thought of the people of any State, and they should consider the relative advantages and disadvantages to the traveling public in districts and cities. It has been the policy of the Texas Highway Department to link the various towns with State highways and to provide for first-class construction along these link routes. In many cases, it has been possible without materially lengthening the line, to serve, directly, the vast majority of towns of more than 200 population and a great many villages of smaller population. However, in serving towns, it is the purpose to provide a direct line, without right angles if possible, and along a street of sufficient width to care for traffic. It is, or should be, the purpose of the engineer in selecting a route through a town to relieve instead of to create congestion. A paved highway connecting with an unimproved street in a city or town or a provision for such connection should satisfy all conditions.

Locating Roads for the Tourist.—It would be quite a different problem if a special frontage tax or assessment were paid by business men dealing in commodities for which there is an incidental demand from the traveling public. Neither the stability nor the growth of a community can depend upon the trade that will come from those who would ordinarily not stop at a place of business unless the highway happened to lead by the door. It would seem

that the authorities in charge of the streets in a small town would prefer to reserve such streets for the citizenship of the community to use for parking their trucks or for stopping their produce wagons. In one Texas town, the main street, which was of considerable width, had been paved and largely paid for by the merchants of the street. A complaint was registered with the Mayor that the people who lived in the town found the street almost blocked with cotton wagons and other traders. The Mayor's reply was that the street was paved for that purpose and that they should adjust their shopping periods to the needs of the community at large, and not try to use the street at the busy period when they could just as well select another time. This is a somewhat radical viewpoint, but the thought is worthy of consideration. In another instance, a filling station operator complained to one of the Texas Highway Commissioners that the highway was being moved from in front of his business to a street two blocks away. The operator exclaimed: "That road has been where it is for fifty years, to my own knowledge," to which the Commissioner replied: "Do you not think it is about time that your neighbor had it for a while?"

One of the chief arguments in registering complaints against the change of location for safety or other reasons within or adjacent to the towns is to say that the State is not building the road for the local people who pay the taxes, but for the tourist. Considerable misinformation has been spread under the name of "tourist trade and traffic." Recently, a Central Texas newspaper contained an editorial stating: "Do not build the road for the tourist but let the taxpayer benefit by having the road." The next editorial a few days after the first said: "Let the traffic pay for the road and relieve the property owner." Within a few days a slogan was published in the same paper: "Make our city the tourist headquarters of the South." These varying comments appeared within a period of seven days.

Webster's dictionary defines a tourist as "one who travels from place to place for pleasure or culture." A citizen of Texas, for example, might leave his home on the morning of a certain day and travel as a tourist along a State highway through many cities and towns; and then, overnight, his status might be changed so that on his return home he would belong to whatever classification might be given him, depending on his rights as a traveler or the contributions he might make for the construction and maintenance of State highways. Such a traveler in Texas might, and very likely would, pay an *ad valorem* tax to build State highways or lateral roads in his home county. He would also, or rather should, pay an *ad valorem* tax on his automobile. He will pay a license fee distributed between the State and County for the maintenance of State highways and, finally, a gasoline tax. When such a traveler leaves his home county for pleasure or culture and travels across his own State, or any other, should he have no consideration given him in that his journey might be made as safe as possible and as direct? Consider, also, for example, a traveler from some other State who might enter the State of Texas and travel for pleasure or culture. In a material way this visitor is required to pay a gasoline tax and, if for no other reason, he is entitled to consideration. State highways should be bought and paid for by the traffic using them and any local

funds used in construction should be refunded. Consequently the maximum consideration should be given the traveler in making locations. He should have the option of proceeding on his journey along the most practical route, and if he desires to visit the business districts of any of the towns along the highway a way should be provided for him in case the route does not lead directly through. No particular distinction except for statistical purposes should be made in the classification of traffic. The motorist is not a class and does not pay taxes with a class for the reason that the vast majority of the Nation's tax-payers in all lines use the Nation's highways.

The Influence of Financing on Location.—The day of the *ad valorem* tax for highway construction is passing. In 1929 the Texas Highway Department recommended to the State Legislature that provision be made for adjustment between the various counties and reimbursement of expenditures for State highways, and this issue failed of submission by a very small margin. There is no doubt that State highways should be built from sources of revenue other than property taxes, and this principle has already been put in effect by a number of progressive States. A study of the accomplishments of such States shows that they are well in the lead of others in the status of their road program. The method of financing State highways should and does have considerable bearing on the location, but even with a portion of the construction cost being paid from an *ad valorem* tax in a road district or in a county, no property owner should claim any special rights to the traffic along a certain road since he pays no special privilege for such traffic.

It is only fair that when a hard surfaced highway has once been constructed along a certain street in a town or into a larger town as far as the laws will permit, that such highway or road should be continued and maintained by some agency; but the owners of business houses along such a route have no right to say that the general traveling public cannot, with finances belonging to the general public, construct such additional highways or by-passes as will enable it to proceed with safety.

The writer agrees with a recent editorial entitled "Getting Away with Antiquated Ideas."* The author deplored the conservatism of many highway executives in clinging to conceptions of road building that were in vogue during the age of horse-drawn vehicles. To salvage the old roads is often not worth while because it is costly in terms of human life.

Conclusion

State highways should be located for present and anticipated traffic requirements so that—whether he be tourist (if there is such a classification), business man, or farmer—the user of the highway can proceed along his route and accomplish his purpose or his duty or receive his pleasure or culture with ease, directness, safety, and economy. Any plan or system of location which does not take these features into account is running contrary to sound public policy. There can be no denying the fact that such public policy, in laying out a great system of highways and lateral roads, must necessarily prescribe a practical directness for the mass of traffic. It must provide more conve-

* *Highway Engineer and Contractor*, Vol. 20, April, 1929, p. 28.

niences for traffic originating locally or bound for local destinations. With the adoption of a plan whereby the revenue for State highways comes from State sources, the solution of this problem becomes easier and the "dog will begin to wag the tail" instead of the contrary, which has been the case in many highway locations.

Highway engineers should take every opportunity to acquaint the public with facts regarding the economy of highway location and to lay before it for inspection the ledger entries on the debit and credit side regarding the operation of a State highway. They should always have an eye for the future and if it is not possible to lay out a plan for economical routing when first construction is obtained, the plan should be so broad as to anticipate future demands.

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HIGHWAY LOCATION: PRACTICAL CONSIDERATIONS*

By A. R. Losh,† Assoc. M. Am. Soc. C. E.

Synopsis

The purpose of this paper is to present some of the major problems of highway location. There are administrative as well as physical limitations which determine in a large measure the work of the engineer on location. It is fundamental that the ultimate purpose of the road be kept in view and the location made to accomplish that end. Highway location does not include merely lines, grade, and construction costs, but traffic as related to towns and cities, other highways, and the general needs of the public. Too much location "for present needs" has been followed in the past with resulting loss, under relocation, to the public and to the individual citizen along the highway as well. Insufficient right of way for the protection of the highway is a serious handicap to its future usefulness.

Introduction

It is recognized that the detail problems of location depend on the peculiar conditions of the individual project. It is equally true that there are general problems on roads of the same class, but these may be decidedly different for roads dissimilar in their uses. Thus, different governing conditions apply in the location of roads for sectional and interstate traffic than for local roads serving small agricultural communities. Roads located primarily for parkways, pleasure drives, and for recreational purposes where scenic considerations are of major importance, will naturally impose conditions differing from those of the other classes. There is, however, one fundamental principle which should be the controlling feature in any road location. Briefly, this is that the resulting location should be adequate for present and prospective future needs of the highway. Present needs are readily determined; future needs require the exercise of judgment and foresight.

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The reconstruction and improvement of roadway surfaces and structures can be readily justified from prolonged usage or for needed increased facilities in the same way that the rebuilding and remodeling of homes, office buildings, mills, and other structures, but the relocation of a large portion or all of the road does not present a similar case. This change of location involves a substantial loss of the right of way, roadbed, and structures, as well as the expense of their reproduction on the new line. It is difficult to find suitable explanation or justification for the building, by the present generation, of many of its main highways three times and on three different locations.

It is not to be understood that the mistakes in location in the past rest solely upon the highway engineer; this is not the case. Many of the earlier improved roads were not located; they merely followed land lines or existing rights of way of older roads. The engineer was given little or no opportunity to develop a suitable location. Road funds were provided by local governmental units and the roads were placed so as to be acceptable to the governing boards or commissions. Most of the present lines of travel were originally only local roads, and the subsequent traffic development was not anticipated. This condition no longer exists except for the local roads. The main highways are financed by the road user at large, by the State, and by the Federal Government. The engineer should approach the location problem with the future uses of the road in mind and should endeavor to provide a location which will be suitable for this purpose. This will include—in addition to alignment, grades, stream crossings, and similar engineering work—provision for future development of the highway and its protection from encroachment by industries and other utilities. Briefly, the mere located line without adequate right of way and roadside protection will not be sufficient.

It will be the writer's purpose to discuss the major problems of locating State and Federal Aid highways. The engineer has received very little assistance on highway location from engineering textbooks. The subject is discussed only briefly if at all, and then usually on the assumption that the existing road will be followed with only minor changes in alignment. The location of highways purely as transportation lines is not the usual practice. This is not a condition readily attainable, nor is it strictly desirable on public highways. The service of the project in the highway plan as a whole is the consideration which should be followed.

Designing for Present or Future Conditions

There are still some who would consider present highway building as primarily of local importance and for the immediate local needs, it being proposed at some future time to build a system of through highways on more direct lines. Such a line of reasoning has been applied to a considerable extent in the past with the following results:

(*a*) The location of important through highways has been materially affected by purely local considerations.

(*b*) Frequently, local and through traffic were combined where a separation would be preferable.

(*c*) When reconstruction for the through traffic is undertaken it is exceedingly difficult to make adequate relocation.

(*d*) When the indirect line is used a number of years subsequent development results in increased property values which may be seriously impaired by substantial changes of location.

(*e*) Right-of-way changes resulting from relocations are usually more expensive than if made under the original construction, due to increased property values and damages to improvements.

(*f*) The value of the old road in terms of right of way, roadbed, surfacing, and structures, is a loss unless the road can be used advantageously for local purposes. Even then, it must be considered that 1 mile of main highway costs as much as several miles of serviceable local roads and there is a loss in changing a main road to a local one.

(*g*) The roadbed structure consolidated and compacted by years of use is lost for reconstruction purposes.

It is this line of reasoning which would make the work of the engineer follow the development of the country rather than to lead and aid materially in that development. It is "the line of least resistance", of temporary expediency, and of future changes with attendant losses. The aim of the engineer should be to provide a location which will remain as a major line of highway transportation and which is so planned as to meet the requirements of the future. A location of this character approaches the "permanent road feature" as nearly as is humanly possible.

Restrictions on Location

On all highway projects there are administrative controls or restrictions which determine at least the general location of the project, or the route of which it is a part. Thus, the termini of the project are predetermined and the intermediate location may or may not be fixed. These restrictions will occur under the following classifications:

1.—Major control points of the Federal Aid System determined by the Secretary of Agriculture and the State Highway Departments and subject to modification by joint action.

2.—State highways and control points designated by State Highway Commissions under legislative authority.

3.—Control points for State highways fixed by legislation.

4.—County or road district bond issues used for a part of the construction cost may require a definite location to be followed.

Under Conditions 1 and 2 there are opportunities for a general engineering study before the controls are specified and also for a revision of controls based on further engineering investigations. Condition 3 may or may not be based on engineering investigation, or it may permit some departure from the established route at the discretion of the State Highway Commission. Condition 4 presents the greatest difficulty because frequently, where local funds are contributed, the detailed location is fixed primarily to serve local wishes. The writer has known of cases in which the increased cost of these features placed by local influences was greater than the local funds provided. For example, in Jefferson County, Oklahoma, county funds were provided to aid in the construction of U. S. Highway 81 on a location determined by the county officials. This location involved three more railroad intersections, one additional bridge, and some increase in distance over a location selected by the State Highway Commission and the United States Bureau of Public Roads.

This system of financing has been responsible for most of the unsatisfactory locations in the Southwest during the fifteen years since 1914. Where it is necessary to continue this system of financing in fairness to all parties concerned, the location should be determined in advance of the local arrangements for financing.

Even under Conditions 1, 2, and 3, there is too great a tendency to follow existing lines of improvement with relatively unimportant control points established; under Condition 4 this tendency often reaches the point of affecting the detail of location. Regardless of how the conditions are imposed they may in effect restrict the work of the locating engineer to mere details of design. Administratively, controls are necessary and should be general in character and limited to the following conditions:

(*a*) A town or city on or near the general line of the route between adjacent control points of the route.
(*b*) A town or city of sufficient importance to be a terminal point for the traffic passing between the adjacent controls.
(*c*) A junction point, preferably near a town or city, of several highways for the distribution of traffic.
(*d*) Topographical features based on engineering investigations.

Within the limitations established as controls the engineer should determine, as far as practicable, upon an economic location, considering at their relative weights both through and local traffic. This selection will be the line which will provide the long-run least cost, including construction, maintenance, and traffic operations. The shortest line will usually be the economic location for the reason that practically all items of cost except earthwork and structures vary directly with the length of the line. In a country of broken topography the cost of earthwork and structures may be much less on the longer line, but the other items of construction, and of maintenance and operation, will usually determine the location. Local traffic can generally be served more cheaply by lateral spurs from the main line than by attempting to give direct service.

One of the usual problems before the engineer is to determine whether or not to swing the main line from its direct course in order to give highway service to an intermediate point, such as a small town or community. Theoretically, the problem can be solved by estimating the cost of construction, maintenance, and operation for each of the two possible layouts; that is, the cost of the main line direct with a lateral spur and cost of the main line on indirect location through the intermediate point.

A similar problem is that of combining two or three highways for some distance out from the city in order to reduce construction mileage. While economically desirable, frequently this involves the questionable practice of encouraging traffic congestion at junctions and terminals.

Studies of State highways show a substantial volume of traffic outside the purely local class. Thus, the California studies of 1922 show the average length of truck hauling to be 31 miles and that of passenger buses, 41 miles. The Pennsylvania studies show that 43.8% of the State highway traffic travels 50 miles or more, and for the Ohio study this figure is 30.3 per cent. Similar

data are not available for the State highways of the Southwest, but it is believed that there is a larger percentage of long-distance movement.

Traffic of this character is benefited more by the possible economic operation over the whole line than by the detail location through small communities with which they may have little or no contact. This provides the basis for considering the needs of local and through traffic separately. It is recognized that grades, character of road surface, alignment, traffic hazards, and conditions causing loss of momentum, use of brakes, and extra wear on the machine are all factors affecting adversely the cost of traffic operation, but the data are not available in a form to be readily used.

Effects of Topography and Grades

In working out the detail location further economies can be made by taking advantage of topographic conditions, soils, available local material, or railroad facilities for importing materials. Approximately 4 000 tons of materials are required per mile of 18-ft. concrete pavement. An increased dead haul of 1 mile will increase the cost of the road about $1 000 per mile. In addition, there will also be the cost of moving construction equipment on and off the job as well as the cost of hauling structural materials. The use of long radius curves reduces the total length of line and the amount of surface widening. For a change in direction of 90°, a curve of 1 000-ft. radius gives a line 214 ft. shorter than a 500-ft. radius. To widen the curve of 500 ft. radius would require about 938 sq. yd. of additional pavement.

Topographical conditions affect the details of location and, in some instances, are major controlling factors. All soils which are unsuitable for subgrades, such as shifting blow sands and rock classifications, are to be avoided if practicable. Overflow sections require high embankments with costly structures, and, for this reason, valley locations are frequently undesirable. Stream crossings must be selected with the view of reasonable permanency of the banks, suitable foundations, and required span length, and they must be within the general location. Grades and curves adjacent to bridges should be reduced to the minimum. It is desirable to have a tangent of at least 150 ft. and a light or flat grade at each end of a major structure. These conditions permit the driver to adjust his machine into the bridge alignment. In the past, grades have probably received more consideration than other features. Considering the motor vehicles now in use some sacrifice of grade can be made to improve alignment. Short grades as steep as 8% can be climbed readily by modern motor vehicles, but continuous grades of more than 6% are objectionable. Tangent profile grades are not particularly advantageous, due to the headlight glare, and do not justify increased expense. A rolling profile fitting the topography is less objectionable for night driving.

Grade separations at railroad intersections are desirable, but not always possible financially. When grade crossings are adopted the line should be located so that future separation can be developed, and in all cases approach curves and grades should not restrict sight distance.

Restricted areas in and near cities are frequently obstructions to direct highway location. This includes parks, golf courses, institutional reservations,

cemeteries, railroad yards, and air ports. Generally, it is impracticable to pass through these areas and the location must be carried around them. The location should give access to railroads and air ports, but should be sufficiently removed to permit full operation of all facilities without congestion. A location directly adjacent to the railroad may be congested by trucks loading and unloading at the station and also by industrial spur tracks being constructed across the highway. Location along the railroad is usually adopted because good alignment is obtainable in this way and because the right of way is not difficult to secure. It has the additional disadvantage of permitting development on only one side of the highway whereas if removed ¼ mile or more from the railroad, property on both sides of the highway would develop.

One of the major problems of the engineer is to provide suitable locations into, through, and around towns and cities. It is necessary to provide suitable connections into the street system of the city. It is highly desirable also to have a direct through connection, even if it may be congested by city traffic, in order to take care of through traffic making a temporary stop in the city. A belt line, or by-pass location, is equally desirable for the through traffic to avoid the city congestion in reaching its destination and to permit the sorting or segregation of traffic for different locations.

The desirable condition for the urban approach would be connections which would permit the distribution of traffic to:

(*a*) The retail section.
(*b*) The wholesale and industrial sections.
(*c*) Railroads and air ports.
(*d*) The several residential sections; and
(*e*) To other highways by direct connections, outside the city proper.

Many town and city officials believe that the highway should be considered as a continuation of "Main Street" out into the country and that it should be carried through the principal thoroughfares of the city. This is especially true of cities of less than 10 000 population and the smaller the municipality the more insistent it is as a rule that the highway pass through the main streets. The larger cities have their local traffic problems and as a rule are glad to secure any relief possible by routing through traffic over outlying streets or through the suburbs. In a great many cities, however, highways are taken directly through retail sections where the streets are congested with electric street cars, motor vehicles, and pedestrians. There are numerous street intersections, sharp curves, grade crossings, congestion due to parking, and other hazards of city traffic. The through traffic is delayed and local traffic congestion is increased with possibilities of accidents multiplied. Too many towns and cities are placing restrictions on through traffic by means of speed ordinances, stop signs, signals, and other regulations. These measures in most cases are necessary for the local traffic, but the through traffic can better operate without them and outside the city. Table 1 gives information on several highways through cities of the Southwest.

The development of proper approaches to cities with arterial routes for traffic to and from the city and belt lines for traffic distribution requires the co-operation of State, county, and municipal authorities. It also requires a

careful traffic study and analysis, upon which to base present and future plans for improvements. The Plan of Highway Improvement in the Cleveland Regional Area is an outstanding example of this character of work. As a result of this survey the City of Cleveland and its contiguous territory have developed a plan of major road and street improvements for ten years ahead and may proceed with individual projects, knowing their final position in the broader plan.

TABLE 1.—DATA ON STATE HIGHWAYS IN CITIES.

City and highway.	Miles in city.	Sharp Curves.		Railways (All Classes).		Street Intersections.		Legal speed, in miles per hour.
		Number.	Total degrees.	Number of tracks crossed.	Route used by street railroad, percentage.	Control signs, signals, police.	Total number.	
Dallas, Tex.:								
U. S. 80	8.5	12	735	19	12	15	89	10 to 20
U. S. 75	7.6	8	870	26	16	16	119	10 to 20
Fort Worth, Tex.:								
U. S. 80	10.9	6	390	9	51	12	123	12 and 18
U. S. 81	9.5	11	1 150	27	77	7	111	12 and 18
Houston, Tex.:								
U. S. 90	8.5	14	780	9	4	13	110	8 to 20
Little Rock, Ark.:								
U. S. 65	4.5	4	525	7	19	4	40	15 to 35
U. S. 167	3.3	2	180	7	6	4	35	15 to 35
U. S. 67	5.0	6	515	7	15	4	56	15 to 35
New Braunfels, Tex.:						....		
U. S. 81	3.2	9	635	1	0	6	30	20
Oklahoma City, Okla.:								
U. S. 77	6.2	6	510	21	9		70	25
U. S. 66	4.15	3	330	4	40	4	42	25
Purcell, Okla.:								
U. S. 77	1.2	4	360	4	0	1	13	....
San Antonio, Tex.:								
U. S. 81	10.0	14*	1 200*	8	28	13	84	10 and 20
U. S. 90	8.6	9	845	23	55	12	95	10 and 20
San Marcos, Tex.:								
U. S. 81	2.2	6	390	0	0	....	32	20
Tulsa, Okla.:								
U. S. 66	9.1	8	591	22	4	7	97	8 to 25
Waco, Tex.:								
U. S. 81	4.2	3	360	25	33	7	39	15 and 20

*Other routes give less curvature, but more traffic congestion.

Increase in most of the speed limits now in force (1929) appear inevitable. The engineer should anticipate this with provisions in his location to meet the conditions. Greater sight distance on horizontal and vertical curves is beneficial; 600 ft. is not excessive. Where this cannot be obtained extra width of surfacing is desirable. In the level and rolling country of the Southwest, engineers are providing horizontal curves of 4° and less. This makes extra width unnecessary, very little superelevation is required, and line length is reduced. The older practice of placing changes in alignment at the top of hills has been discontinued. It is considered a hazard to traffic. Other conditions being equal, curves should be placed where they may be seen readily by the approaching driver.

Right-of-Way Features

To a great extent engineers have allowed right-of-way considerations to influence location. This is especially true in regions where the section-line road system prevails. It is seldom that on rural highways more than 80 ft. of width is secured, which amounts to approximately 9½ acres per mile. Considering the cost of State highways at about $30 000 per mile there could be a justified expense of $3 000 per acre for a proportional reduction in length of construction. This does not take into consideration any savings in maintenance and operation costs.

The width of right of way needed depends on the future requirements of the road and also the means for its protection from encroachment and exploitation. Few States have adequate laws regarding right-of-way acquisition and roadside protection. A Texas statute permits the State to obtain 100 ft. for a State highway, which ordinarily is ample. Oklahoma may obtain an easement for any width "for highway purposes." There is nothing, however, to prevent the abutting property owners from building garages, service stations, and other roadside establishments on the property line, and utilizing a part of the highway for the transaction of their business.

Until such time as adequate legislation is enacted placing roadside development under the supervision of highway authorities, the engineer should use such means as are available to protect and preserve the highway for its complete utilization. To this end the writer offers five suggestions:

(1) Acquire sufficient right of way for future development of the highway.

(2) Obtain extra widths at all road intersections and other strategic points where it is likely that roadside enterprises would be established.

(3) In making acquisitions it will usually be possible to retain in the right of way the narrow strips, gores, and other small parcels of land cut off from the original tract. These parcels may be utilized for highway purposes in the future or for roadside planting and beautification. In any event they are not available as sites for billboards and roadside establishments.

(4) Through areas of natural beauty either obtain in fee or protect with long-term leases, several hundred feet extra width on each side of the right of way.

(5) The interest of civic organizations and public-spirited citizens should be encouraged and their aid secured to obtain the desired ends.

Public Considerations

It is apparent to every one that, aside from the commercial features of highways, there are social and political considerations which cannot and should not be disregarded. Roads are for public usage and must be located, constructed, and maintained with that end in view. Whether for business or for pleasure the public travels to definite points of interest and attraction. Schools, churches, and Courts are the mainstays of the Commonwealth. The public must have access to these institutions by highways. Rural mail and express deliveries, transportation of farm products, and rural community life, while usually served directly by local roads, are affected by their connection with the main highways. There are patriotic shrines in every section of the

country which should be made accessible from the highways. National and State parks and forest preserves are visited annually by thousands of citizens and, with improvements in the roads, these properties will be used by many more for recreational purposes.

Every governmental unit should be connected finally with the country at large. The writer has always believed that the larger unit of government has this obligation to its constituent parts: The Federal to the State, the State to the county, and the county to its towns and communities.

These considerations of schools, churches, Courts, and other conditions vitally affecting the citizenship of the Nation cannot be weighed in terms of economics or engineering, but must be taken under the "General Welfare" clause of the fundamental law.

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RELOCATING LAND SURVEYS IN TEXAS*

By B. F. Williams,† M. Am. Soc. C. E.

Synopsis

This paper describes the problems encountered by engineers in Texas who are engaged in relocating old survey lines. The conflicting influence of Spain and England on the land laws are discussed, and the modern equivalents of old and obsolete units of measurement are given. The paper includes also an interesting anecdote concerning the monumenting of the 100th meridian.

The principal emphasis, however, is directed toward outlining some practical rules to govern the engineer on relocation surveys. Of what use to the surveyor is the evidence pertaining to annual rings of old trees? How is it possible to obtain direct evidence of the location of a timber line? How was the land cleared? The answers to these and other important questions are given in this paper.

Original Marks

It is a well fixed principle of law that in relocating surveys it is the duty of the surveyor to follow the boundary lines as laid on the ground in the original survey. "Move not the ancient land marks" is an old legal maxim given by Moses in Deuteronomy. Land boundary litigation has burdened the Civil Court dockets since the beginning of private ownership of the soil, and many wars between kings, princes, barons, and lords have been fought over disputed boundaries.

Indefiniteness in description and inaccuracy in laying down the original boundaries cause most of such litigation. As an example the following is cited from the archives of the Texas General Land Office:

"In the same Town on the same day, month, and year that precedes, Don Juan de la Garza and Don Pedro de Salvana, the Surveyors, the former on the part of His Majesty and the latter on that of the Town, came into our presence and said: That in conformity with the Act made known to them on

* Presented at the meeting of the Surveying and Mapping Division, Dallas, Tex., April 25, 1929. Written discussion of this paper will be closed in the March, 1931, *Proceedings*.

† State Reclamation Engr., Austin, Tex.

the third, they proceeded to mark out the boundaries assigned to the Town on the south, and using the cord composed of fifty Mexican varas recognised by the Commissioners present, they stretched the same one hundred times (beginning at the center of the Town) which make five thousand Mexican varas or one complete league, with which they arrived at the point known as La Cantera, which they marked out; with as many more they reached a vale with woods which are transitable and which is named the stopping place called Senor San Jose; with as many more to the gully at the foot of the hills of Los Quatitos and they dedicated it to our Senor of San Juan; with as many more to the stopping place called Agua-pas and dedicated it to our Lady of La Sotemano; with as many more to a hollow in front of Abra del Quemado which they dedicated to our Lady of Dolores; with as many more to the margin of the River Blanco which they dedicated to our Lady of Loreto, and they are the six leagues which in accordance with the order communicated to them they have surveyed and marked out, each league for itself on said course. And from said plain they returned to the Town and directing themselves to the North from its center they stretched the cord one hundred times and arrived at the Palitos Blancos and dedicated it to our Lady of Santa Ana; and with another hundred cords commencing on the other bank of the Rio Grande del Norte (for the reason that but a few varas remained between the said league and the river) they arrived at a hill which they dedicated to San Joachin and marked out the second league; with as many more to an opening in the hills (Abra) called El Yachupin which they dedicated to San Francisco de Borja; with as many more to the end of the said opening in the hills to the place called San Lorenzo; with as many more to the opening in the hills of San Yldepfonso which they dedicated to Santa Theresa de Jesus; and with as many more to a hill which is on one side of the stopping place called San Yldepfonso which they dedicated to La Santissima Trinidad, which six leagues they surveyed and marked out in the same manner as those above specified, always following the said course North. And this is what they did in compliance with their duties, and without having any thing to add, they signed with us and those of our assistance.=Palacio.=Lizos Ossino.=Juan Jph de la Garza Falcon.=Pedro Joseph Ygnacio de Salvana.=of Assistance, Duran.= of Assistance Federico Losada."

Such deficiencies as these are the causes of the greatest difficulties to the surveyor in making a relocation. The boundaries of a survey are usually a series of connected straight lines having given lengths and bearings, and designated natural or artificial objects, marks, and monuments; or they follow the meanderings of streams or divides of water-sheds. The natural marks are streams, mounds, hills, rocks, trees, etc. The artificial marks are crosses, blazes, hacks, drill holes, stakes, letters, figures, mounds, pits, a pile of buffalo bones, a cow's head, or other conventional signs and characters on standing timber, rocks, ledges, or on the ground. In order to retrace the lines of a survey it is necessary to have the description of the original boundaries made by the surveyor who laid them down. This description, called the field notes, is usually found in the archives of the General Land Office or in the records of deeds in the county where the property is situated. It is also necessary to know the laws, rules, regulations, and conditions under which the survey was made.

Texas Laws

All early surveys in Texas were made under Spanish and Mexican Laws and regulations. The territory within the present boundaries of the State—

under the discoveries of Columbus, the conquests of Cortez, and the explorations of Coronado, De Soto, and others—was claimed by Spain until the successful Mexican Revolution of 1821. This claim, however, was disputed by the French on the basis of La Salle's explorations in 1684. Therefore, the land laws of this State have been influenced by the Spanish and French laws, which have a common origin in the Roman Civil Law. After the Texas Revolution of 1836 the principles of the English Common Law took precedence over the Spanish and French influences, but the land titles granted by Spain and Mexico were recognized and ratified by the Republic and the State of Texas. Prior to the Texas Revolution, title to about 35 000 000 acres had been granted.* Texas was attached to and administered as part of the Mexican States of Coahuila and Tamaulipas from 1821 to 1836, when it became an independent republic. In 1845, it was admitted into the American Union of States. By a provision of the act of admission, Texas retained title to all her public vacant land, and the State has administered these lands chiefly for the benefit of railroad development and the endowment of a public educational system. The United States has never owned or administered the public lands of the State and, therefore, the Federal system of land surveys has never been adopted.

The Arabs and Moors who helped to develop the Spanish laws and customs, having established governments in the arid and semi-arid territories of Arabia, Northern Africa, and the Iberian Peninsula, impressed upon the Spanish law rules for laying out surveys of land, giving each tract or grant a front on a stream of permanent water wherever possible. The Spaniards followed this policy in North America.

The streams of the State of Texas all have a general southeasterly course; therefore, all Spanish and Mexican surveys make an angle with the meridian and seldom does the width northwest and southeast exceed one-half the length northeast and southwest; and the statutes now provide that in laying surveys along a stream they shall have a stream front of not more than one-half the square. Stephen F. Austin, impresario of one of the earliest colonies, in a letter of instruction to Elias Wightman, a surveyor, directs that surveys shall be laid along the streams, giving each a water front, and if any land could not be covered, but should be left vacant, it would be on the prairies where the land had no value.

Measuring Units

The Spanish and Mexican laws prescribed the units of land measurement and the methods of laying out surveys and of marking the lines and corners. The linear unit is the "vara" (in the Austin collection of documents frequently called "bar"), which is equal to 33⅓ in. and is defined as 3 "geometrical" ft.† or "pie". The term, "pie", is a Spanish unit of length, corresponding roughly to 1 ft. In Castile it compares with the Old Roman foot, as 923 is to 1 000.‡ The vara had different values at different dates and in different

* Wooten's, "History of Texas."

† "Lamar Papers," Edited by Harriet Smither, Vol. V, p. 129.

‡ See, Velazquez Spanish Dictionary.

countries. For example, in Peru, it equals 33 in.; in Spain, 33.384 in.; in Argentina, 34.092 in.; and in Portugal, 43.7007 in.* When the Americans began to make surveys in Texas, each surveyor had his own idea of the length of the vara. Stephen F. Austin† prescribed the length as 33⅓ English in. and this value has become the standard.‡ The geometric foot was generally used in the Sixteenth Century. DeMorgan§ gives its value as 9.8 in. and Locke§ as 10 in. The units of area are the square "pie", square vara, "labor", and "league" or "sitio". An English acre is equivalent to 5 645.376 sq. varas. A labor contains 1 000 000 sq. varas, or 177.13 acres. It is 1 000 varas square. The league is 5 000 varas square and contains 25 labors, or 4 428 acres. Many of the Spanish surveys contains a league and labor, or 26 labors, which is equal to 4 605 acres. Five leagues or sitios is a hacienda. In the theory of the Spanish laws and customs the labor is a farm, or an area upon which to labor and to produce crops. In contradistinction to the labor, a league is land upon which to conduct ranching and stock-raising operations.

It is also interesting to note a peculiar custom of ancient origin practiced in Latin countries in connection with granting and delivering land. The patent contains a certificate over the signature of the administrative officer that upon a certain date he took the grantee by the hand and led him around the land and pointed it out, and the grantee went upon the land and pulled herbs and threw stones and proclaimed in a loud voice that he was the owner of the said land. However, a similar custom was practiced by many ancient peoples.

In the Spanish laws, natural objects received preferential importance over artificial marks as monuments and witnesses to the boundaries in the specifications for laying out surveys, and in numerous opinions, the State Courts have uniformly held that natural objects, other elements of the proof of the locations of the survey being equally certain, shall be given superior importance. In relocating surveys the Courts consider four elements in the following descending order of importance: Natural objects, such as rivers, creeks, mountains, etc.; artificial objects, such as marks on trees, marked lines, stakes, etc.; course and distance; and quantity. These rules are founded on reason, experience, and observation, and the surveyor must approach the problem of relocation with them well in mind, because he may be, and frequently is, required to defend the accuracy of his work before the Courts. While the surveyor must know the law, he has done his duty when he has found all the facts relating to the original boundary, relocated it upon the ground, and reported his findings of fact to his principal, or to the Court. In many cases he will be unable to harmonize the description of the original boundaries with the facts he finds on the ground.

Two cases will illustrate the point. A surveyor returned field notes to the General Land Office for 116 sections of land along the right bank of the Canadian River. His certificates state that the work was done on the ground, that the boundaries were marked by natural and artificial objects, and that

* See, Fourteenth Edition, Encyclopedia Britannica.
† Baker's "Life of Austin."
‡ "Lamar Papers," Edited by Harriet Smither, Vol. V, p. 129.
§ Century Dictionary, under "Geometrical Foot."

he began at a definite point, and progressed toward the west giving each survey a river front of ½ mile. Each succeeding survey begins at the northwest corner of the preceding survey on the right bank of the Canadian River. In making a relocation on the ground it was found that one of these sections would cross the river twice, and a part of it, at its northwest corner, would lie on the left bank of the stream and all succeeding surveys in the series would lie on the left bank if joined to the northwest corner of the preceding survey.

During a lawsuit involving the location of some of the surveys in this series the surveyor testified that the only field work done was the taking of the few bearings on the course of the river, while an assistant kept watch at the top of the cliff to give warning of Indians. After taking these bearings on the course of the river and assuming that it continued indefinitely on the same course, he went into the safety of his office and wrote out the field notes. When asked why he did not make the surveys on the ground as certified, he said, "It would have taken a company of United States soldiers to have kept the Indians off of me while making these surveys, and the land was not worth it." In Fig. 1 the solid lines show the location as called for in the field notes; the dotted lines show the actual river with the surveyed tracts as they would lie in relation to it.*

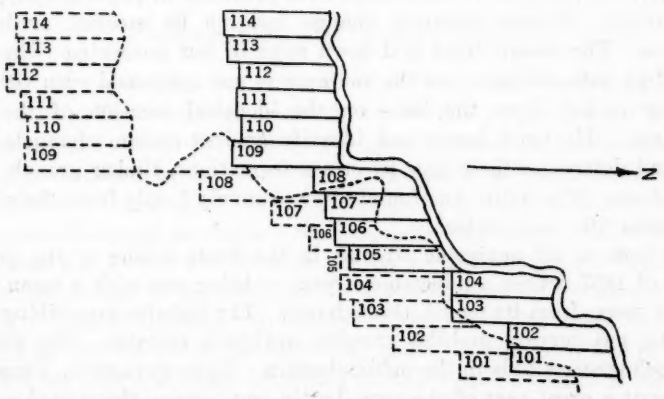

FIG. 1.—SECTION OF THE CANADIAN RIVER IN THE TEXAS PANHANDLE, SHOWING EFFECT OF INACCURACY IN OLD SURVEYS.

In 1886 or 1887, a district or county surveyor, in order to locate tracts of school land in the Panhandle, sold by the State, ran a connecting line from a monument on the 100th meridian west (at right angles to that meridian at the initial point) for 50 or 60 miles to the school surveys, and located the tracts in that manner. It is plain that he did not cross the next meridian at right angles, and his corners were located several hundred feet too far south. These corners conflicted with the adjoining alternate sections previously located and patented to a railroad company for constructing its lines, and the owners had the surveyor enjoined by Court proceedings. The result was that a settle-

* Sanborn *et al. v.* Gunter *et al.*, Southwestern Reporter, Vol. 20, p. 72.

ment of various conflicting claims became necessary and by co-operation with the State a resurvey of all the railroad and alternate school sections in that part of the State was made.

It is interesting to note that the position of the initial monument on the 100th meridian, located by Jones and Brown in 1859, by observations of lunar culminations, when checked in 1892 by Messrs. H. S. and H. C. Pritchett and T. U. Taylor, M. Am. Soc. C. E., for C. A. Culberson, Attorney General of Texas, to secure evidence in the trial of a suit brought by the United States against Texas to recover the territory in Greer County, was found to be 3 797.3 ft. west of the 100th meridian. In 1929 a new monument was set in correct geodetic position by Mr. S. S. Gannett under the directions of a United States Supreme Court decision, and the true 100th meridian was run and monumented.* This adds a strip of territory about 134 miles long, about 880 ft. wide at its north end and 4 040 ft. wide at its south end, to the eastern edge of the Texas Panhandle, over which Oklahoma had theretofore exercised sovereignty. The Legislatures of the two States are now (1929) adjusting the civil affairs of this strip.

Qualifications of Surveyor

In making investigations of natural objects called for in a description of a boundary, the surveyor is confronted with problems in physiography, geology, and ecology. Stream channels change location by erosion, avulsion, and accretion. The Courts have laid down rules of law governing titles to lands affected by such changes, but the surveyor is not concerned with titles. It is his duty to lay down the lines on the identical location of the original boundaries. He must locate and identify ancient stream channels and terraces and determine their ages by strata formations, timber growth, and collateral facts. The writer has found river channels ¾ mile from their locations as recorded fifty years before.

The beds of all navigable streams in the State belong to the public. A statute of 1837 defines a navigable stream as being one with a mean width of 30 ft. or more, from its mouth throughout.† The statutes prescribing methods of laying out surveys prohibit crossing navigable streams. The effect is to leave such streams beds in the public domain. Many streams in Texas are dry throughout a great part of the year, but in wet seasons the wetted perimeters are several hundred feet long and the width between the cut banks of the freshet flows makes them navigable under this statutory definition. It appears that surveyors understood the definition of a navigable stream to mean a 30-ft. width of water and gave no importance to the width between the cut banks of the flush flow. These flashy streams are located in the arid or semi-arid regions, which at the time the surveys were made, were sparsely settled. The work was done under great difficulty of transportation and accessibility of supplies, and with great dangers from savage marauding Indians. It was usually done in the warm, dry season, and the surveyors in many instances found these streams dry. They located the surveys across the streams in

* See, Rept. of Samuel S. Gannett, Boundary Commr., Washington, D. C.
† Article 5302, Texas Rev. Civ. Statutes, 1925.

violation of the statute, if it is assumed that the Congress of the Republic meant to prescribe a width between cut banks and without regard to the width of water. The Supreme Court of the United States, however, in the Texas-Oklahoma Boundary Case laid the common boundary between these States on the south cut bank of the Red River as located under the treaty of 1819 and as subsequently changed by erosion and accretion.* The surveyor is frequently called upon to locate the public lands in the stream beds of the State, and the work requires highly technical knowledge and skill. It is necessary to locate the original boundaries of the adjoining surveys and determine the changes made by the forces of the stream.

An investigation of artificial objects requires no less skill and experience. A fair knowledge of botany, plant anatomy and physiology, and ecology is required. Many efficient, capable, and skillful surveyors have no definite or organized knowledge of these sciences and probably have never heard or seen the names of some of them; they are students in them nevertheless. A little native wit and common sense, that is, sense about common things, supplies a great deficiency in knowledge of organized science. There is probably no technical branch of work that requires more common sense and self-reliance than surveying. Species, genera, habits, and methods of growth of trees must be known and ages of marks must be determined in many cases by inspection.

Identifying Survey Lines on Timber

Consider, for example, the importance of knowledge concerning annual rings on trees. The exogenæ, as the name of this great division of plants indicates, enlarge outwardly, each year's growth showing in cross-section as a ring. The cambium layer, which is next under the bark, is the only growing part. Since the inner wood does not change by growth, a mark put on it remains until destroyed by decay. Scar tissue is deposited in the matrix; the wound heals, but a scar will show on the bark for many years. Neither the character nor the age of the mark can always be determined by the scar on the bark, and it may be necessary to cut away the wood and count the annual rings back to that on which the mark was made. Marks sixty-five or seventy years old have been positively identified in this way. Often it is difficult to determine definitely the presence of a mark on a tree; the lateral enlargement of the trunk distorts it, and on an exposed side it may be more or less obliterated by disintegration of the bark; therefore, it may happen, and frequently does, that two competent and credible witnesses will disagree as to the presence of a mark.

The field notes describing a boundary usually give the sizes and genera of the trees called for, and sometimes they will include descriptions of timber lines and character of flora. While magnitude of size varies directly with magnitude of age, the surveyor must not assume that all large trees are very old or that all small trees are very young and, therefore, refuse to examine small trees for old marks. In 1920, in making a relocation, the writer found a cedar

* State of Oklahoma *v.* State of Texas, U. S. Intervenor, U. S. Repts., Vol. 256, p. 70.

elm which was marked in 1842 as a witness to a corner. The locating surveyor gave the size as 6 in. in diameter. In 1920, the diameter was 8 in. On another occasion, a post oak, 3½ in. in diameter, was compared with a sweet gum, 18 in. in diameter. The oak had 52 annual rings, while the larger sweet gum had only 60 annual rings.

Often it becomes highly important to determine the locations of old timber lines on some of the early surveys. If the notes call for a timber line at a given distance from a corner, or for timber at the corners, and these locations are found in fields or on prairies, it is necessary to determine whether the conditions called for existed at these locations when the survey was made. To make the investigation pits are dug 2 or 3 ft. in the ground. By careful examination of the vertical banks of these pits the forms of tree roots can be traced by the presence of root bark or coloration of the earth long after all trace of the wood fiber of the roots has perished. Most hardwood trees have large tap roots, penetrating deep into the substratum of clay. These roots persist long after all surface traces of the trees are gone, and even after the roots have decayed completely, their forms can be easily traced by the residual humus and the surface soil washed and settled into their clay moulds.

The writer's experience indicates that the pioneer surveyors ran most of their lines long, and that the bearings vary on the ground considerably from those given in the field notes; but in making a relocation of an old survey, if the surveyor will use a light, open-sight compass with an ordinary Jacob staff he will have little difficulty in following the marked line.

Duties to Clients

A proper approach to a difficult relocation is very important. The surveyor is usually called in after a controversy has arisen to which his client is a party. The client expects the surveyor to find the facts in his favor, but the only interest the surveyor should have is in finding the facts. He should make his position clear to the contending parties and with the record before him approach the problem very much as a detective investigates the mysteries of a great crime. He should run down clues and test out theories until some one or more facts have been established. Then by applying the descriptive matter of the record, he should find corroborative facts and make out the complete case. He should observe the rules of legal evidence, because they are the results of experience from time so remote that the memory of Man "runneth not to the contrary." Hearsay testimony without corroborative facts cannot be accepted, except as clues.

When the investigation is finished, the surveyor must be certain of the results beyond a reasonable doubt. If he has found sufficient facts to justify it, he should relay the boundary on the location of the old one regardless of interests or conflicts. On the other hand, if the facts are not sufficient, he should not make a relocation, but should report to his client the conflicting evidence, giving the client the benefit of all clues, theories, and facts found during the investigation. Doubtful relocations and conflicting interests can be settled by agreements or adjudicated by the Courts.

Conclusion

In making relocations the surveyor must have a definite conception of his duty. If the field notes state that the original surveyor began at a definite point and ran the lines on given bearings for given distances; that he marked certain objects and monuments at the initial, terminal, and intermediate points; and if those objects can be found on the ground and can be identified by the marks, the lines joining them fix the original boundary regardless of course, distance, or quantity of land. Therefore, it is the duty of the surveyor to find and identify the objects called for and lay the relocated boundary on the ground accordingly. The surveyor is employed to find and report the facts. His duties do not require him to become an advocate of his principal, but it is his privilege and duty to defend the facts set out in his report.

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THE BEHAVIOR OF A REINFORCED CONCRETE ARCH DURING CONSTRUCTION

Discussion*

By Searcy B. Slack, M. Am. Soc. C. E.†

Searcy B. Slack,‡ M. Am. Soc. C. E. (by letter).§—The discussions have brought out a number of valuable points and have added materially to the paper in other ways. In general, the subjects raised require more research work rather than discussion, but in a few cases it is evident that some further explanation of the paper should be made.

Mr. Gilkey points out‖ that the stress measurements on the surface of the concrete appear to be rather high. Perhaps enough emphasis was not given in the paper to the statement that these stresses were computed from strain measurements made on the surface of concrete, and that strain (or change in surface dimensions) might be caused by a great many factors other than stress.

Mr. Gilkey further points out¶ a slight discrepancy between Conclusion 6 and Fig. 13 (*b*). A better wording for Conclusion 6 would be as follows: "The temperature of the concrete after being placed in the arch rib increased rapidly during the first few days and then fell gradually." As a matter of fact, all temperature curves except the ones published, show that the temperature had reached very nearly the maximum at a period of 1½ to 2 days. In the case of thinner sections of the arch, the fall in temperature did begin after a period of about 18 hours.

In his discussion¶ Mr. Fuller takes exception to Conclusions 2 and 4. It might be implied from Conclusion 2 and the text of the paper that a fall in

* Discussion of the paper by Searcy B. Slack, M. Am. Soc. C. E., continued from September, 1930, *Proceedings*.

† Author's closure.

‡ Bridge Engr., State Highway Dept., East Point, Ga.

§ Received by the Secretary, September 29, 1930.

‖ *Proceedings*, Am. Soc. C. E., February, 1930, Papers and Discussions, p. 365.

¶ *Loc. cit.*, p. 366.

temperature of from 115° Fahr. to about 30° Fahr. was suggested for use in design. This drop would be from the maximum temperature during time of set to the minimum reasonably to be expected in the concrete. To suggest this much fall in temperature was not intended. There is much evidence indicating that there is re-adjustment of stresses between the concrete and the steel, which would tend to relieve initial stress; hence the use of the full temperature range is not justified. The amount of fall in temperature to be used in design should be based on further study and reseach work, but it is the writer's opinion that somewhat more than one-half the total fall indicated would nearly represent existing condition; that is, a fall of 50 to 60 degrees.

While a re-adjustment of stresses between concrete and steel due to the fall in temperature probably does take place, it is a little difficult to imagine that a gradual rise in temperature above the mean reached between the concrete and steel will cause an actual reversal of the former stresses.

Conclusion 4 is obviously open to question as are all stress measurements which are based upon strain converted into stress. This is particularly true when the measurement extends over a considerable period of time.

Mr. Turner* brings up the point that "error in the interpretation of extensometer and telemeter readings follows the assumption that change of form of the steel invariably represents deformation attributable to elastic stress induced by applied loads." Gauge points used for telemeters and strain-gauges throughout this work were located on straight sections of the steel and were at least 2 ft. from the nearest bend. Under these conditions it is believed that strain measurements on the steel bars may reasonably be assumed to represent stress.

Mr. Turner states† further that the test blocks which were used in these experiments were too short to develop the tension which would occur in longer specimens. If the tension in the steel in the early stages is due to a difference in the coefficient of expansion of the concrete and steel (which seems to be the most logical explanation of the initial tension), the length of the specimen would have no particular effect on the amount of tension, provided the length was great enough to develop bond fully between the concrete and the steel.

After the reports were prepared on this work, additional measurements were made on steel bars in Test Blocks Nos. 1 and 2 referred to in the paper.‡ These measurements were made three months and five months after the blocks had been cast. Those made at three months indicated that practically all the initial tension in the steel had been relieved, and at five months there was a very slight compression in the steel. These further measurements bear out Professor Fuller's statement§ of a re-adjustment taking place between the concrete and the steel.

Mr. Strahan has suggested‖ some interesting applications of the equipment used in this work, and it is to be hoped that some of the research suggested by him may be done.

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* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 1036.
† *Loc. cit.*, p. 1035.
‡ *Loc. cit.*, November, 1929, Papers and Discussions, p. 2290.
§ *Loc. cit.*, February, 1930, Papers and Discussions, p. 367.
‖ *Loc. cit.*, September, 1930, Papers and Discussions, p. 1657.

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EFFECT OF TURBULENCE ON THE REGISTRATION OF CURRENT METERS

Discussion*

By B. F. Groat, M. Am. Soc. C. E.

B. F. Groat,† M. Am. Soc. C. E. (by letter).‡—In connection with this paper it will be of considerable interest and value to discuss a subject that bears directly upon the averaging of current-meter readings taken in a vertical velocity plane, or—what amounts to the same thing—to state those results giving "rules" similar to those of Cotes, Newton, Simpson, and others, but especially formulated to supply averages in the form of the arithmetical mean. The observations and tables by Messrs. Yarnell and Nagler present opportunities for applying these suggestions.

All hydraulic engineers are familiar with the "two-tenths eight-tenths depth method" of averaging two readings to obtain the mean velocity in any vertical. This rule is based on the hypothesis that the actual vertical velocity curve approximates the arc of a parabola. The rule is exact for a parabola with its axis parallel to the current. It is also exact for a straight-line velocity gradient. It is known that, in particular cases, the actual velocity curve may depart widely from the arc of any parabola. However, if it is possible to state a rule, or rules, based on several point-velocity readings at various depths, such that the arithmetical mean represents the average for a curve of higher order than that of a parabola, the error of the mean can be reduced to an increasing extent as the number of such points is increased. In current-meter work, the appropriate arithmetical mean has the peculiar advantage that the extreme ordinates are not at the ends of the base line (water surface and bottom), as is the case with the usual rules of mensuration. The qualification, "appropriate", is made because there is an infinity of arrangements of ordinates of equal weight.

* Discussion of the paper by David L. Yarnell and Floyd A. Nagler, Members, Am. Soc. C. E., continued from September, 1930, *Proceedings*.

† Cons. Engr., Brookline, Mass.

‡ Received by the Secretary, August 7, 1930.

A general formula for mensuration was rigorously deduced by the writer in a paper, entitled "Mean Value of the Ordinate of the Locus of the Rational Integral Algebraic Function of Degree, n, Expressed as a Weighted Mean of $n + 1$ Ordinates, and the Resulting Rules of Quadrature", read before the American Mathematical Society, at Providence, R. I., on September 12, 1930. It furnishes not only the well-known formulas of mensuration (Newton's rule, Simpson's rule, etc.), as special cases, but also supplies, as other special cases, the rules conforming to the specification for current-meter readings (as previously stated) at various depths in the vertical-velocity plane.

These results may be stated briefly under eight headings, as follows:

(1) The "two-tenths eight-tenths depth method" is misnamed;
(2) Range of validity of mensuration rules;
(3) Weight ratios for "two-tenths eight-tenths depth method";
(4) Arithmetical mean of three ordinates;
(5) Arithmetical mean of four ordinates;
(6) Arithmetical mean of five ordinates;
(7) Arithmetical mean of six ordinates; and
(8) Arithmetical mean of ten ordinates.

(*1*).—*The "Two-Tenths, Eight-Tenths Depth Method."*—This method is misnamed. While the error caused by this inexactness may be moderate in some cases, yet such a name should be discarded because it is misleading. The correct relative depths (relative distances from the ends of the base line in mensuration) for the two velocity readings, when measured from either the water surface, or the bottom, are exactly, and, respectively, $\frac{1}{2} \mp \sqrt{\frac{1}{12}}$, or, approximately, in decimals, 0.211325 and 0.788675. It should be observed that this not only represents the true area of a parabola through the two points, but also the area bounded by a straight line passing through the two points, the base line, and end ordinates (water surface and bottom).

(*2*).—*Range of Validity of Mensuration Rules.*—The foregoing relation is a general truth in mensuration; if n points lie on the locus of a rational, integral, algebraical equation of lower degree than $n - 1$, then the true formula for n points will not only be a true rule for the locus of a rational, integral, algebraical equation of the $n - 1$th degree, but, also, for all loci of equations of lower degree than $n - 1$. Therefore, the rule for a parabola works on a straight line; the rule for a cubic works on a parabola and on a straight line; the rule for a quartic works on a cubic, a quadratic, and a straight line, etc.

(*3*) *Weight Ratios for "Two-Tenths Eight-Tenths Depth Method."*—The rule for the so-called "two-tenths, eight-tenths depth method" results by making the weight of the middle (mid-depth) ordinate of the parabola equal to zero by properly locating the other two ordinates, so that Cotes' rule (the basis of Simpson's rule), 1:4:1, is replaced by 1:0:1, thus giving equal weights to two ordinates. It was shown in the paper before the American Mathematical Society that the ordinates may be located anywhere and their respective weights calculated; or *vice versa*, the weights may be chosen and the ordinate locations calculated. This method is applied to curves of higher order, thus

obtaining further simple rules for averaging ordinates (point-velocity readings in a vertical velocity curve), some of which are indicated in Table 9.

TABLE 9.—SPECIAL QUADRATURE RULES FOR THE MEAN VALUE OF A FUNCTION, DERIVED FROM THE GENERAL RULE.

Heading No. (1)	x (x'). (2)	y (y'). (3)	z (z'). (4)	r. (5)	Ordinate spacing. (6)	Remarks. (7)
	(4) ARITHMETICAL MEAN OF THREE ORDINATES ($n = 3$)					
4	$\frac{1}{2} \mp r$	.....	$\frac{1}{2}$	$\sqrt{\frac{1}{8}}$	r	.....
	0.1464466	.....	$\frac{1}{2}$	.....	.....	.....
	0.8535534	.....	.....	.....	.....	.....
	(5) ARITHMETICAL MEAN OF FOUR ORDINATES ($n = 4$)					
5	$\frac{1}{2} \mp r$	$\frac{1}{2} \mp \frac{1}{3}r$	.....	$\sqrt{0.15}$	$\frac{2}{3}r$	.....
	0.1127017	0.3709006	.....	.....	.....	.....
	0.8872983	0.6290994	.....	.....	.....	.....
	(6) ARITHMETICAL MEAN OF FIVE ORDINATES ($n = 5$)					
6	$\frac{1}{2} \mp r_1$	$\frac{1}{2} \mp r_2$	$\frac{1}{2}$	$\sqrt{\frac{1}{4} - \frac{7 \mp \sqrt{11}}{48}}$	$r_1 - r_2 : r_2$	Ordinate spacing not uniform.
	0.0887513	0.3127298	$\frac{1}{2}$	0.416248743	.....	r_1 in Column (5).
	0.9162487	0.6872707	.....	0.187270705	.....	r_2 in Column (5).
	(7) ARITHMETICAL MEAN OF SIX ORDINATES ($n = 6$)					
7	$\frac{1}{2} \mp 5r$	$\frac{1}{2} \mp 3r$	$\frac{1}{2} \mp r$	$\sqrt{\frac{1}{4} - c}$	$2r$	.....
	0.07920706	0.2475242	0.4158414	.....	.....	.....
	0.9207929	0.7524758	0.5841586	0.08415859	.....	.....

Under Heading (7) in Column (3), $c = z - z^2$. Thus, r is a function of the coefficients of a cubic. The rules tabulated determine an average ordinate in terms of the arithmetical mean of n properly located ordinates.

In Table 9 the column headings, x (x'), y (y'), and z (z'), refer to the relative values of the abscissas corresponding to ordinates symmetrically located in pairs with respect to the center of the base line. Thus, $x + x' = 1$, etc. When there is an odd number of ordinates, z and z' (Column (4)) become identical as the relative abscissa of the mid-ordinate.

Abscissas are relative distances from either end of the given base line (surface, or bottom, for current-meter velocities). When a great number

of averages, or resulting areas, are to be determined from drawings, or profiles, a diagram of radiating lines may be drawn on a separate piece of tracing cloth, so as to touch the ends of a base line within the limits of the tracing cloth and also to intersect the base line at intermediate points relatively distant from the ends by amounts proportional to the abscissas concomitant with the rule to be used. The diagram with its base parallel to the given base, and with its end lines touching the ends of the given base, will locate immediately all the intermediate points, along the given base line, at which ordinates are to be erected. The arithmetical mean of these n ordinates is the average value which, when multiplied by the length of the given base, gives the area enclosed by the profile, the base line, and two ordinates erected at the two ends of the given base on the drawing. The two end lines on the diagram may be used to locate convenient consecutive ends for a succession of bases into which a long line is to be divided.

These consecutive bases need not necessarily be equal, but had better be chosen so as to secure greater accuracy, depending upon the nature of the profile to be examined. The end ordinates do not enter the arithmetical mean. If they did, the result would not be so accurate, since the relative weights of end ordinates, for symmetry about the center, are less than unity. The rules give the exact area under the curve of a rational, integral, algebraic curve of degree, $n - 1$ (that is, of a degree less by 1 than the concomitant number of ordinates stated). If the ordinates are drawn to any curve of more or less smooth shape, the arithmetical mean of the ordinates is an approximation to the mean value of the ordinate of that curve, the degree of approximation increasing with the number of ordinates (and with the degree), as previously explained. The rule for n ordinates not only gives the exact area for degree, $n - 1$, but also for all lower degrees.

Universal Applicability of the Rules.—There can be no doubt that proper use of Rules (4) to (8), inclusive, will lead to greater accuracy in reducing field work, whether in water measurement or other classes of engineering. The accuracy of the formulas can be checked by applying them to curves for which the proper ordinates have been calculated. Use the radicals, r, because the symmetry causes them to disappear.

Limitation on the Use of Weddle's Rule.—When the base is divided into six equal parts by seven ordinates, one at each point of division, including the ends of the base, Weddle's rule states that the weights are, respectively, 1, 5, 1, 6, 1, 5, and 1, but it is not exact for seven ordinates to a curve of the sixth degree. The correct weights for this case are, respectively, 41, 216, 27, 272, 27, 216, and 41; the sum equals 840.

The arithmetical mean of ten ordinates representing generally the area under the locus of an equation of the ninth degree would furnish a most facile method for practical use, since it requires nothing more than addition and shift of a decimal point. The weights for ten ordinates which divide the base into nine equal parts are, beginning at either end, respectively, 2 857; 15 741; 1 080; 19 344; 5 778; 5 778; 19 344; 1 080; 15 741; and 2 857; total, 89 600.

(*8*).—*Arithmetical Mean of Ten Ordinates.*—Until the more exact rule for the arithmetical mean of ten ordinates is adduced, the arithmetical convenience of ten ordinates, together with an improvement in accuracy, can be had by using the rule for five ordinates on a base divided into two halves, just as one applies Simpson's (Cotes') rule to two, or more, sub-bases. The relative locations of the ordinates for this purpose are: 0.04187563; 0.15636465; 0.2500000; 0.34363535; 0.45812437; 0.54187563; 0.65636465; 0.75000000; 0.84363535; and 0.95812437. The arithmetical mean of ten ordinates thus located is the desired average to about seven places of figures; that is, accurate for two arcs of quintics which, together, pass through all ten points.

Tables 7* and 8† present the essence, and show the advantages, of the method of "aggregates" or the "statistical method".‡ In any good gauging section, the ratio of discharge to the aggregate revolutions per unit time (or aggregate velocities, if preferred) for readings taken at given points is a determinate function of the elevation of the water surface. When the discharge is constant, as in a mill-race, the function is constant. The usual statement that accuracy cannot be made less than 2% is too conservative. If one takes care to eliminate unknown influences as far as possible, including such conditions as rapidly fluctuating stages, and then determines proper constants by means of proper aggregates, the discharge can be gauged within a fraction of 1 per cent. Most of the difficulties with current meters have arisen when trying to harmonize varying readings when the aggregate used was altogether too thin. Engineers must distinguish carefully between accidental errors and systematic, or constant, service.

Figs. 23§, 24‖ and 25¶ show that there is some kind of relation between such widely separated conditions of rating as those at Iowa and in the rating tank of the University of Michigan. It does not seem that there should be any very close agreement between ratings in turbulent water and ratings in steadily flowing water, even if the average angle of approach and average direction should be the same in the two cases. This is best illustrated by the differences visible in Fig. 24. However, it is interesting to know that the relation is closer than might be suspected.

* *Proceedings*, Am. Soc. C. E., August, 1930, Papers and Discussions, p. 1413.

† *Loc. cit.*, p. 1416.

‡ "Characteristics of Cup and Screw Current Meters; Performance of These Meters in Tail-Races and Large Mountain Streams; Statistical Synthesis of Discharge Curves," *Transactions*, Am. Soc. C. E., Vol. LXXVI (1913), pp. 819, 838.

§ *Proceedings*, Am. Soc. C. E., August, 1930, Papers and Discussions, p. 1409.

‖ *Loc. cit.*, p. 1410.

¶ *Loc. cit.*, p. 1411.

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BALDWIN FILTRATION PLANT, CLEVELAND, OHIO

Discussion*

By E. Sherman Chase, and J. W. Ellms, G. W. Hamlin, A. G. Levy, and J. E. A. Linders.†

E. Sherman Chase,‡ M. Am. Soc. C. E. (by letter).§—The writer has been impressed not only by the size of the Baldwin Filtration Plant, but by the general excellence of its design and construction. It is gratifying to have so complete a description of the plant available in permanent form.

The use of the hydraulic jump‖ for bringing about the mixing of the coagulating chemicals with the water is one of the interesting features of the plant. This method has the obvious advantage of simplicity and absence of mechanical equipment. On the other hand, there is a loss of head involved of approximately 2½ ft. which would be largely avoided had mixing tanks equipped with stirrers been installed. Furthermore, stirring devices capable of being operated at variable speeds permit somewhat greater flexibility of operation than flumes of fixed dimensions.

The application of the aluminum sulfate in solution rather than in the dry form is a departure from what seems to be the present trend in filter plant practice.¶ It is probable, however, that in a large plant with an adequate laboratory staff, the adjustment of the alum dose can be regulated more exactly by the method adopted than by dry feed. Economy in the use of coagulants is of substantial merit where large quantities of water are treated daily.

The adjustable weir plates on the wash-water troughs are novel features and the wisdom of this provision has apparently been substantiated in practice as shown by the experiences described by W. C. Lawrence, Superintendent of Filtration at the Baldwin Plant.** According to Mr. Lawrence difficulties in securing proper cleaning of the sand grains during filter washing were

* Discussion of the paper by J. W. Ellms, G. W. Hamlin, A. G. Levy, and J. E. A. Linders, Members, Am. Soc. C. E., continued from September, 1930, *Proceedings*.

† Authors' closure.

‡ Cons. Engr. (Metcalf & Eddy), Boston, Mass.

§ Received by the Secretary, August 14, 1930.

‖ *Proceedings*, Am. Soc. C. E., February, 1930, Papers and Discussions, p. 209.

¶ *Loc. cit.*, p. 250.

** "Variable Wash Water Rates with Changes of Water Temperatures," *Journal*, Am. Water Works Assoc., Vol. 22, No. 2 (1930).

overcome by increasing the rates of vertical rise with increased temperatures of the raw water. Had fixed overflow weirs been provided at the lowest elevation available, it would have been difficult to have readily increased the rate of wash without undue loss of sand.

The advantage of high-velocity wash appears to have been shown elsewhere. This is particularly true at the filter plant at Greenwich, Conn., and has been demonstrated experimentally at Detroit, Mich.* It seems probable that in the future more liberal provision for high wash-water rates will be made in filter-plant design than has been the general practice in the past.

The characteristics of the filter sand closely follow the usual specifications. Experience in the Greenwich plant indicates that the requirements for a low uniformity coefficient may be unnecessary in the case of rapid sand filters. At this plant, six of eight filters were provided with sand having an effective size of 0.36 to 0.37 mm. and a uniformity coefficient of 1.48 to 1.55, whereas, in two other filters, the sand had an effective size of 0.37 mm. and a uniformity coefficient of 4.65.† Operation records have demonstrated equally satisfactory results with the two sands. The reason for equal performance is probably due to the fact that the hydraulic grading of the sand by washing results in the finer sand being brought to the surface of the bed where most of the purification is accomplished.

In closing, it is interesting to note that the City of Cleveland has expended about $7 000 000 for the purification of a supply which, as late as 1910, was considered fairly satisfactory even without chlorination. This fact illustrates the increasing demand on the part of the public for water supplies of the highest degree of purity.

J. W. Ellms,‡ G. W. Hamlin,§ A. G. Levy,‖ and J. E. A. Linders,¶ Members, Am. Soc. C. E. (by letter).**—The engineers who have so kindly discussed this paper have brought out a number of points in the design which are worthy of further comment.

The question asked by Mr. Van Loan†† relative to the operating results with the hydraulic jump mixing flumes, may be answered by calling attention to the results obtained from five years of successful operation. These flumes have efficiently mixed the alum solution with the raw water, and the flocculation following the jumps has been entirely satisfactory.

The use of an alum solution instead of dry feed does depart from the general trend of present-day practice, as mentioned by Mr. Chase.‡‡ The writers are fully aware that this is the case, but experience has shown that

* "Studies on the Washing of Rapid Filters," by Messrs. Hulburt and Herring, *Journal*, Am. Water Works Assoc., Vol. 21, No. 11 (1929).

† "The New Purification Plant of the Greenwich Water Company." *Journal*, New England Water Works Assoc., Vol. LXII, No. 4 (1928).

‡ Engr. of Water Purification and Sewage Disposal, Dept. of Public Utilities, Cleveland, Ohio.

§ Engr. of Constr., Div. of Water and Heat, Dept. of Public Utilities, Cleveland, Ohio.

‖ Engr. of Constr. and Surveys, Div. of Water and Heat, City of Cleveland, Cleveland, Ohio.

¶ Engr. of Design, Dept. of Public Utilities, Div. of Water and Heat, Cleveland, Ohio.

** Received by the Secretary, September 29, 1930.

†† *Proceedings*, Am. Soc. C. E., August, 1930, Papers and Discussions, p. 1457.

‡‡ See p. 2037.

accurate and uniform, and consequently economical, application of the coagulant has been obtained at the Baldwin Plant.

The preparation of a dilute alum solution of uniform strength by the method used at the Baldwin Plant, namely, the dissolving of a weighed quantity of alum by the upward flow of a measured volume of water, is an extremely simple process; and when the solution is properly agitated before using, does not require laboratory supervision. The writers are firmly convinced that the continuous uniform measurement of a liquid is more accurately and more easily obtained than the continuous measurement of solid particles.

The experimental work* upon which the design of the hydraulic-jump flumes was based, conclusively demonstrated the thorough mixing action produced by this method. In this work the exploration of the jump with Pitot tubes showed water currents moving in all directions within the jump. The writers cannot conceive of any form of mechanical stirring which would produce "a more commensurate initial jarring of the water", as stated by Mr. Jenks,† than that obtained in the hydraulic jump.

The Baldwin Plant was designed, in the section providing for the preliminary treatment, on the basis of two steps: First, for a practically instantaneous, thorough, and intimate mixing of the chemical coagulating solution with the water; and, second, for a gentle agitation of the entire flow of the treated water over a period of time to permit of the agglomeration of the floc. The purpose of the first step was simply one of mechanically mixing the alum solution with the water, and the element of time is immaterial provided thorough mixing is effected. The elements of time and velocity, however, do enter into and play a very important part in the second step of the process, namely, the agglomeration of the colloidal particles into a floc formation. The second step in the process is accomplished at the Baldwin Plant in the large conduit, equipped with sets of "over and under" baffles, which connects the hydraulic-jump mixing section with the coagulation basins. The length of time required for this secondary agitation depends upon the character of the water being treated, and the quantity of coagulant added. In experimenting with Lake Erie water, the writers found that this time period need not be longer than 5 or 6 min., with a velocity of 0.75 to 1.00 ft. per sec. A pin-point floc is characteristic of Lake Erie water treated with alum as the coagulant. The usual small quantity of suspended matter in Lake Erie water may account for the small size of the floc formed.

The writers agree with Mr. Jenks† that the loss of head through the hydraulic jump is comparable with that in the conventional type of baffled mixing chamber; but they seriously question whether the same degree of thorough and rapid mixing of the chemical with the water is obtained in the baffled chamber.

The design of the Baldwin coagulation basins was based on a theoretical velocity of flow and a definite period of detention. It was considered desirable

* "The Hydraulic Jump as a Mixing Device," by A. G. Levy and J. W. Ellms, Members, Am. Soc. C. E., *Journal*, Am. Water Works Assoc., January, 1927.

† *Proceedings*, Am. Soc. C. E., August, 1930, Papers and Discussions, p. 1458.

to have an average velocity of 2 to 2.5 ft. per min., and a detention period of 4.5 to 5 hours.

Continuous sludge removal was not permissible at the Baldwin Plant because the sludge could only be removed in the spring and fall months.

More shallow sedimentation basin design than is commonly used appears to have a reasonable basis from theoretical considerations. However, certain practical deductions from observations extending over many years, with many different types of water, lead the writers to believe that efficient sedimentation of suspended matter, with a specific gravity as close to that of water as is produced by coagulating Lake Erie water with alum, requires average depths of not much less than 15 ft. It will have to be proved that in so mobile a liquid as water the velocities in basins of only about one-half this depth are not destructive of efficient sedimentation in the lower strata of water at and near the bottom. The writers believe caution should be used in applying the over-flow rate theory as at present developed. One questionable point is the relationship between total depth and the overflow rate depth. Whatever factor may be used for this ratio would seem to be affected materially by the character of the suspended particles. As variations in temperature of the water will change its viscosity, it does not seem improbable that efficiencies of coagulation basins may be affected by this fact, as noted by Mr. George G. Nasmith.*

The hydraulics involved in the design of a filter plant is of the utmost importance and affects its successful operation. Experience has proved that the greatest care and skill are required in making the computations involved in the various hydraulic problems entering into the proper design of a filtration plant. When these computations are carefully made, it will be found that the actual hydraulic gradient through the filtration plant will be in very close agreement with the theoretical gradient as calculated.

Good design involves the proper velocities in mixing the coagulant with the water, in secondary agitation to produce proper flocculation, in channels and through sluice-gates to prevent breaking up the floc, and in coagulation basins to permit of effective sedimentation. Adequate available head for filter operation and for washing filters should also be provided. Gutter and drain capacities for wash water, when used at the highest rates of flow, are imperative to permit proper cleaning of the filters. Unless all these points of design are carefully considered and properly calculated an efficient plant cannot be constructed.

An additional feature of the purification process at the Baldwin Plant not originally contemplated has been the recent adoption of the ammonia-chlorine process of disinfection by which the problem of chloro-phenolic tastes appears to have been solved. No such objectionable tastes have been noted since the adoption of this process at the Cleveland filtration plants in January, 1930.

The writers are glad to note that Mr. Herron† finds the cost data to be of value and in convenient form for use. The writers believe them to be one of the most important features of the paper.

* "The Trial Filtration Plant, Ottawa, Canada," by George G. Nasmith, *Journal*, Am. Water Works Assoc., August, 1930.

† *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1727.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

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COMPLETION OF MOFFAT TUNNEL OF COLORADO

Discussion*

By A. A. Eremin, Assoc. M. Am. Soc. C. E.

A. A. Eremin,† Assoc. M. Am. Soc. C. E. (by letter).‡—This paper, together with that entitled "Construction Methods of the Moffat Tunnel", by R. H. Keays, M. Am. Soc. C. E.,§ is of much value to the profession; it describes many interesting engineering and economical features of tunneling. Moreover, in his reference to the late David H. Moffat,‖ the author calls attention to the fine public spirit of one engineer in working with untiring activity for the welfare of a city.

The author has stated that observations on the lining of the tunnel have proved the statement made by the Board of Consulting Engineers that if weak rock is held securely it has a tendency to reach a constant distributed bearing, thus insuring permanent safety.¶ This statement is in agreement with the theory of pressure on deeply overlaid tunnels as developed by Kommerell.**

When a tunnel is excavated in a soil of uniform cohesion an arch effect occurs over the bore, and the pressure on the lining will be expressed by the weight of the area of the parabola, *A B C*, in Fig. 13 (*a*). If a side pressure occurs, the loading diagram will be expressed by the shaded area in Fig. 13 (*b*). The curve, *A' C' B'*, is a parabola or, for practical purposes, it may be considered as an ellipse. Since the height, *H*, in Fig. 13, is a function of the cohesive properties of the ground, a comparatively uniform bearing pressure will occur in the arch roof of the tunnel unless it is complicated by some physical or chemical action in the soil.

In discussing†† timber lining, the author refers to the previous statement‡‡ by Mr. Keays that "the decision to try to hold this heavy ground permanently with timber only was a bold one."

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* Discussion of the paper by Clifford Allen Betts, M. Am. Soc. C. E., continued from September, 1930, *Proceedings*.

† Sacramento, Calif.

‡ Received by the Secretary, September 27, 1930.

§ *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 63.

‖ *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 681.

¶ *Loc. cit.*, p. 698.

** Kommerell, "Statische Berechnung von Tunnelmauerwerk"; see, also "Tunneling," by Eugene Lauchli, 1915, p. 104.

†† *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 697.

‡‡ *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 95.

From Fig. 13, it is evident that the pressure on the tunnel lining varies with the size of the tunnel and the arching effect of the ground and that it does not depend on the depth of the overlying mass.

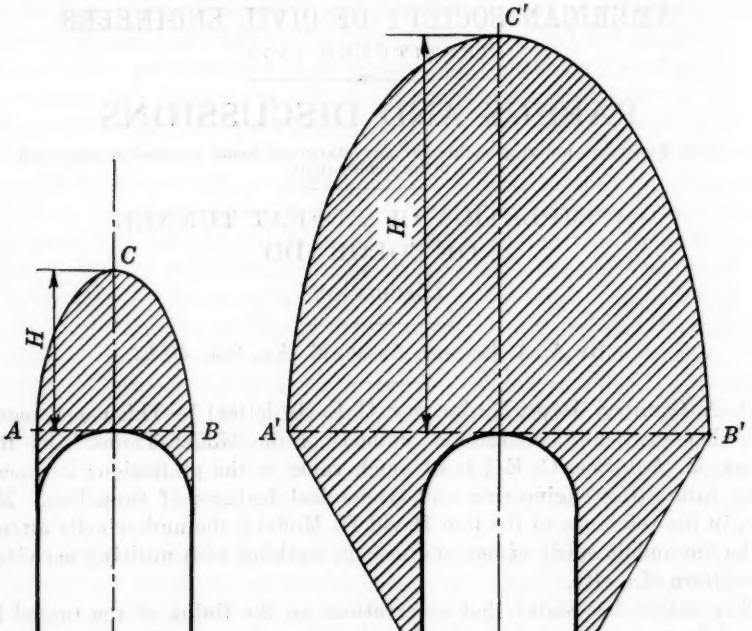

FIG. 13.—DIAGRAMS SHOWING EARTH PRESSURE ON TUNNEL LININGS.

The most common cause of failure in the timber of a segmental arch rib is due to shear at the joints.* Many factors can be blamed for this local development of shear, namely, poor workmanship, local and uneven distribution of swelling in the ground, soft material, unsymmetrical stratification of the ground in relation to the center line of the tunnel, and an insufficient number of segments used in the arch ring.

In actual practice it is very seldom that the soil penetrated by a tunnel is of uniform formation. Usually, it is cut by seams and fissures and shattered by faults. A geological survey is expensive and, therefore, in order to have a greater factor of safety, a structural material with better resistance to shear, and flexural stresses, than timber is usually selected for the lining of deep tunnels. A serious disadvantage of timber lining is the difficulty of preserving it from decay. High temperature, moisture, and chemicals dissolved in the ground are very injurious.

Useful information concerning soil pressure may be obtained from an observation of timber lining left for some time to support the ground before enclosing it in concrete for the permanent lining.

Information on tunneling is scarce in technical literature. Therefore, this paper is especially important to the Engineering Profession.

* *Engineering News-Record*, December, 1927, p. 1039.

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ANALYSIS OF CONTINUOUS FRAMES BY DISTRIBUTING FIXED-END MOMENTS

Discussion*

By Messrs. Robert A. Black, H. E. Wessman, Edward J. Bednarski, and S. N. Mitra.

Robert A. Black,† Assoc. M. Am. Soc. C. E. (by letter).‡—The writer has had occasion to apply the Cross method to the design of simple two-legged bents symmetrically loaded and to continuous beams simply supported, and he knows from experience with what ease, rapidity, and accuracy, indeterminate structures of the type mentioned may be solved. The paper, however, reveals the application of the method to bents of one or more stories subjected to side-sway due either to unsymmetrical or to horizontal loads.

It so happened that the writer was confronted with a problem concerning two-story bents subjected to traction forces and to unsymmetrical loading. The comparative ease and simplicity of the solution afforded by the method described, emphasizes the fact that the subject matter of the paper offers an invaluable tool for the designer. A brief description of the problem and one phase of its solution follows.

In connection with the extension of a new Outer Drive over the Chicago River and of the development of the air-right territory over the tracks of the Illinois Central Railroad and the Michigan Central Railroad Companies, north of Randolph Street and east of Michigan Avenue, Chicago, the South Park Commissioners are required to build certain viaducts. These are known as the Randolph Street, the Field Boulevard, and the Wacker Drive, Viaducts. They all will be two-level structures having, in general, transverse girders at the column bents and longitudinal floor-beams between bents, and they will be built entirely over railroad-yard tracks. Because of certain sub-surface rights granted the Railroad Companies, it is required that the columns of these viaducts be carried down to Elevation —19.0 (Chicago City Datum), at

* Discussion of the paper by Hardy Cross, M. Am. Soc. C. E., continued from October, 1930, *Proceedings*.

† Acting Structural Engr., South Park Commrs., Chicago, Ill.

‡ Received by the Secretary, September 3, 1930.

which point they will rest on caissons. As the top of the curb of the top deck is at Elevation + 47, the columns will be nearly 65 ft. long. They are limited in size in a direction normal to the center line of track to 2.0 ft., including the concrete encasing. Due to the multiplicity of tracks and their varied alignment, to loading platforms and team tracks, and to limited head-room allowances, the bracing which may be used will be confined entirely to small knee-braces.

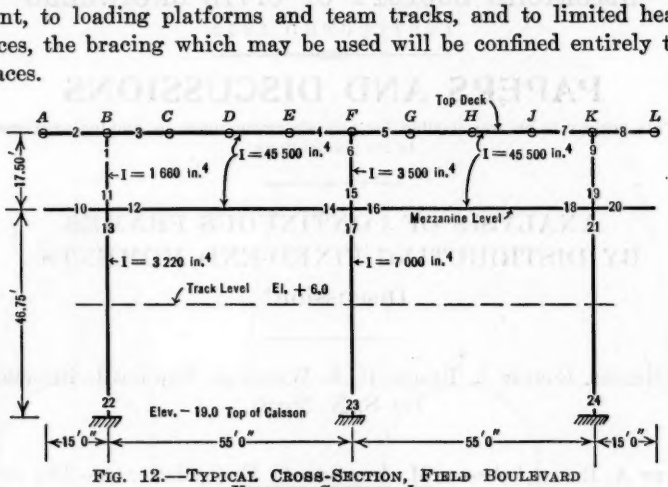


FIG. 12.—TYPICAL CROSS-SECTION, FIELD BOULEVARD VIADUCT, CHICAGO, ILL.

The stiffness of these structures will depend almost entirely upon the slender columns and the rigidity of the connections. Therefore, it is important to know the effect of traction and of vertical loading upon bending in the columns and upon the strength of connections. A series of influence lines was considered to be the best means of studying the columns and their connections in these viaducts.

A typical cross-section of the Field Boulevard Viaduct is shown in Fig. 12. The letters refer to floor-beam points on the top deck and the numbers on the structure refer to points at which moments were computed, that is M_1, M_2, etc. The solution for the ordinate to an influence line with the load on the top deck only, may be outlined in six steps as follows, the unit load in this example being placed at Point C:

1.—With the unit load at Point C on the top deck, the fixed-end moments at B, and F are -7.734 and -2.578, respectively (see Fig. 13). Distribute these moments as demonstrated by the author in Fig. 1,* using a "carry-over factor" of $-\frac{1}{2}$, and considering no sidewise deflection of the bent. Only the final moments, M'_C, are given in Fig. 13.

2.—After the distribution operations, compute the column shears (see Fig. 14), H'^{T}_{C} and H'_{BC}. (The subscripts refer to the top or bottom story with a unit load at Point C.)

3.—Take the same bent, but without the unit vertical load, and apply any convenient set of fixed-end moments to the top-story columns. The only

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 922.

restriction on these moments is that they shall be proportional to the $\frac{I}{L^2}$ of the columns to insure that the deflections at the tops of the columns will be the same. For the present purpose values of 100, 211, and 100, respectively, were selected and applied as shown in Fig. 15. Distribute these moments as before; sum them up and call the final moments, M''. Repeat Step 3, beginning with the lower-story columns. Call these final moments, M''' as in Fig. 16.

FIG. 13.—VALUES OF THE MOMENTS, M'_C, WITH A LOAD AT POINT C, IN FIG. 15, AND WITH NO SIDEWISE DEFLECTION.

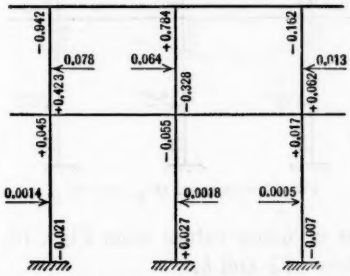


FIG. 14.—COLUMN SHEARS, H'_{TC} AND H'_{BC}, WITH A LOAD AT POINT C, IN FIG. 15, AND WITH NO SIDEWISE DEFLECTION.

4.—From the moments, M'' and M''', compute the column shears in both stories. These shears are designated H''_T, H''_B and H'''_T, H'''_B, respectively. (See Figs. 17 and 18.)

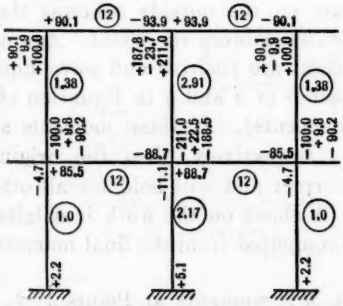


FIG. 15.—MOMENTS, M'', DUE TO DISTORTION IN THE TOP STORY.

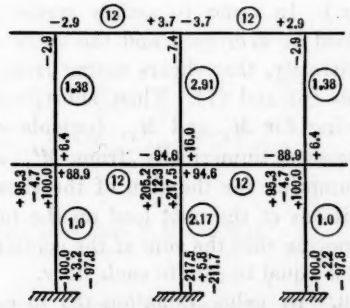


FIG. 16.—MOMENTS, M''', DUE TO DISTORTION IN THE LOWER STORY.

5.—The total shear in each story is made up of the shear due to the vertical load and that due to the assumed moments causing the side-sway, and this total shear must equal zero to satisfy statics; that is, the total shear is equal to H' plus some fraction of H'' and H'''. Therefore,

$$H'_{TC} + a\,H''_T + b\,H'''_T = 0 \quad \ldots\ldots (4)$$

$$H'_{BC} + a\,H''_B + b\,H'''_B = 0 \quad \ldots\ldots (5)$$

or, using the values from Figs. 14, 17, and 18, Equations (4) and (5) become,

$$-0.027 + 42.10\,a - 2.40\,b = 0 \quad \ldots\ldots (6)$$

$$+0.0001 - 0.643\,a + 17.18\,b = 0 \quad \ldots\ldots (7)$$

Solving Equations (6) and (7) simultaneously, $a = + 0.000642$ and $b = + 0.000018$.

The total moments, therefore, at Points 1, 2, 3, etc. (Fig. 12), due to the unit vertical load at C and corrected for side-sway are,

$$M_C = M'_C + a\,M'' + b\,M''' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

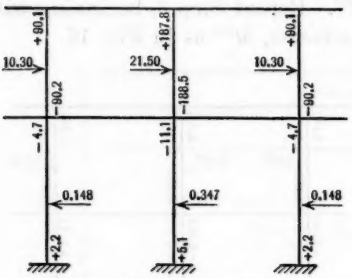


FIG. 17.—SHEARS, H''_T AND H''_B.

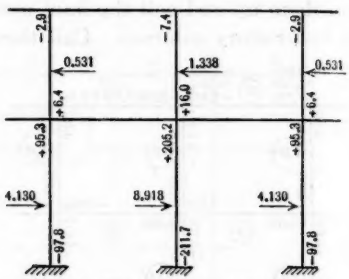


FIG. 18.—SHEARS, H'''_T AND H'''_B.

that is, using values from Figs. 13, 15, and 16, and the previously determined values of a and b,

$$M_1 = - 0.942 + 0.00064 \times 90.1 - 0.000018 \times 2.9 = - 0.884$$

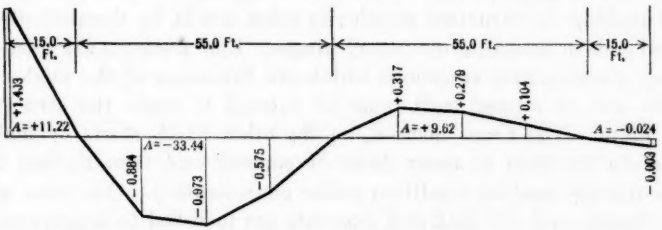
and so on for all other moments, M_2, M_3, etc.

Sign of a.—By inspection of Fig. 14, it is seen that the resultant shear in the top story (0.027 lb.) acts from right to left. (According to the convention used by Professor Cross the shears as shown in Fig. 14 are acting on the lower portion of each column. In this example, all shears have been shown in this way.) In order to satisfy statics the shears on the outside columns then, should be decreased and the shear on the center column increased. Assume, arbitrarily, that shears acting from left to right are positive and solve Equations (6) and (7). Then, substitute these values of a and b in Equation (8), solving for M_1 and M_{11} (outside column moments). If these moments are decreased numerically from M'_1 and M'_{11}, respectively, then the original assumption for the sign of the shears was correct and will hold for all other positions of the unit load on the top deck. A check on the work is obtained by noting that the sum of the column shears computed from the final moments, M_C is equal to zero in each story.

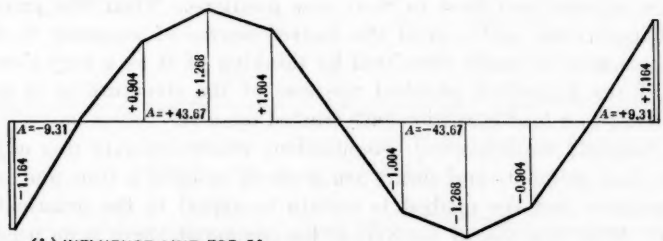
6.—By using Equation (8) to correct all M'_C moments at Points 1, 2, 3, etc., of Fig. 12, the material is derived with which to plot two ordinates (at C and at J) on the influence line for the moment at any one or all of the points shown in Fig. 12. It should be noted that the effect of the load at J on M_1, for example, will be the same as that of the load at C on M_9, the bent being symmetrical, except that the sign of the moment changes when dealing with column moments. (The sign remains unchanged for girder moments.) Examples of influence lines, selected at random, are shown in Fig. 19.

It should be noted, also, that in carrying the problem on for positions of the unit load at other points on either deck, the values of M'' and M''', and H''_T and H''_B, and H'''_T and H'''_B, herein used for Point C, may also be used for the load at all other points.

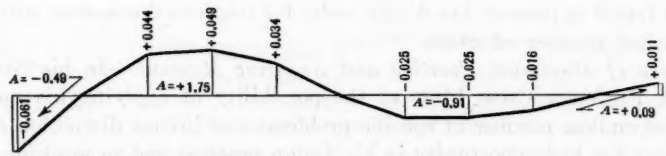
After the load has been run across the top deck, it can be placed on the lower deck and an independent set of influence lines obtained for moments at the same points, 1 to 24, inclusive, shown on Fig. 12. The conclusions and results which may be obtained from these influence lines are obvious and need not be mentioned here.



(*a*) INFLUENCE LINE FOR M_1



(*b*) INFLUENCE LINE FOR M_6



(*c*) INFLUENCE LINE FOR M_{13}

FIG. 19.—INFLUENCE LINES, IN FOOT-POUNDS UNITS, FOR N, M, AND M_{13}, WHEN THE TOP DECK ONLY IS LOADED.

In this example, tractive forces are acting normal to the bent and hence do not affect the moments herein computed. However, in any case where they are an element to be considered, their effect may be determined by Steps 3 to 6, inclusive, because in this case there are no moments, M', and the shears do not equal zero, but are equal to the known tractive force. This alters Equations (4), (5), and (8). Otherwise, the procedure is the same.

In the example given, the moments of inertia of the various members were obtained from a preliminary design, and while their actual values may change somewhat their relative values should remain about constant. Almost without exception, all computations were made on a 10-in. slide-rule. The method has proved accurate, a great time saver, and a valuable aid to intelligent design.

H. E. WESSMAN,* Assoc. M. AM. SOC. C. E. (by letter).†—Professor Cross' method of analyzing continuous frames is the most outstanding contribution made in many years to the field of structural engineering. Although it has for its fundamental basis the geometry of slopes and deflections of elastic members—which is common to all logical methods of analyzing indeterminate structures—it is developed in a much different way. Other methods usually begin by making the structure simple; in other words, by theoretically cutting restrained ends or inserting temporary hinges. End moments are then obtained by solving simultaneous equations which are functions of the angles through which the cut or pinned ends must be rotated to make the structure continuous again. The Cross method, on the other hand, starts by making the structure rigidly fixed at every joint or support and then finding fixed-end moments for any loading condition under consideration. One joint at a time is then released, and the fixed-end moments are modified to secure equilibrium at the released joint. This joint is temporarily fixed again until all other joints have been released and fixed in their new positions. Then, the procedure is repeated again and again, until the desired degree of accuracy is obtained. The process may be easily visualized by thinking of it as a very slow-motion picture of the magnified, physical response of the structure as it gradually adjusts itself to a load condition just created upon it.

The complete mathematical computations which evaluate this adjustment and give final moments and shears are made in so brief a time and in such a simple manner that the method is certain to appeal to the practical-minded engineer. With this tool of analysis at his command, there is no need for the designer to "shy away" from indeterminate structures. There is no excuse for "guessing" moments and shears by means of arbitrary coefficients such as those found in present-day design codes for reinforced concrete, which apply to a limited number of cases.

Curves of Maximum Positive and Negative Moment.—In his concluding remarks, Professor Cross hints of the possibility of applying his method to an almost endless number of specific problems and invites discussion on them. The writer has had opportunity in his design practice and in teaching to compare the application of several methods to some of these problems. He much prefers that of Professor Cross and wishes to illustrate its workability in constructing curves of maximum moments and in analyzing frames subject to lateral displacement. Curves of maximum positive and negative moment have been computed for a four equal-span continuous beam for various ratios of uniform live load to dead load (hereafter called the $\frac{L}{D}$ ratio). In the first case, there is no restraint at the supports from columns or other structural elements. In the second case, corresponding curves are computed for a four-span system having the same span lengths as the first, but in which rigidly connected columns at the supports affect the moments. This case is one in which analysis by previously accepted methods becomes very complex. It is just as easily treated by the author's method, however, as the first case in

* Associate Prof. of Structural Eng., Ministry of Railways Univ., Shanghai, China.
† Received by the Secretary, September 8, 1930.

which there are no restraining columns. A study of the two sets of curves will not only show the effect of column restraint in changing curves of maximum moments for the beams, but will raise some questions about present design codes for reinforced concrete.

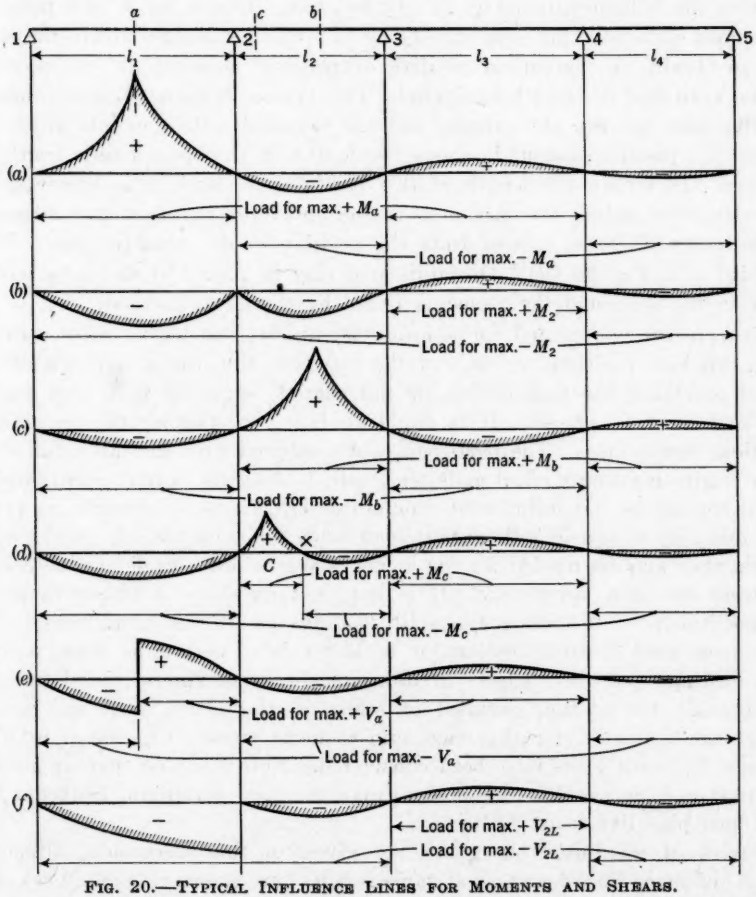


FIG. 20.—TYPICAL INFLUENCE LINES FOR MOMENTS AND SHEARS.

It is necessary to know where to place live loads in order to obtain the greatest effect at any point (or section) of the continuous system. Here is where the influence line performs its chief function. It is primarily a guide for the intelligent placing of live load. Fig. 20 shows typical influence lines for moments and shears at various governing points in a four-span continuous system. They are not quantitative; only qualitative. To most engineers familiar with indeterminate structures, sketching the influence line is only a moment's task. In most cases a sketch is sufficient to tell one where to place the loads for maximum effect. For the governing points of the design, that is, points along the middle one-half of the span, Fig. 20 (*a*) and (*c*), and

points at the supports, Fig. 20 (*b*), the influence lines for moment show that only full span loads are needed. They change sign (from positive to negative) only at the supports. In reinforced concrete design, these points are the only ones that need to be investigated in most cases.

Note the influence line, Fig. 20 (*d*), however. This is for M at a point in the center span near the support. There is a change in sign within the span and to obtain the maximum positive or negative moment at this point a partial span load must be investigated. This type of influence line is common to all points between the support and the so-called critical point, which, in beams of constant section, is about two-tenths of the span length from the support. To obtain the length of this partial span load, it is necessary to find numerical values for only a short section of the influence line adjacent to the point where it crosses from the positive to the negative side. This is Point x in Fig. 20 (*d*). One unit load may be placed at the three-tenths point in the span and the moments found by the Cross method.

The process is repeated for a unit load at the four-tenths point. From these two load positions, enough of the influence line can usually be determined to obtain the load divide, or the partial length of span over which live load must be placed. It is doubtful, however, whether this is of any practical significance. The part-span load combined with the other full-span loads to give maximum effect calls for a split loading that rarely occurs under actual conditions. In reinforced concrete design, there is certainly no practical necessity to consider these part-span loads for moments. In steel girder design, they may be needed for the longer spans in order to determine cover-plate cut-offs more accurately. It is largely a question of judgment as to the possibility of obtaining the split loadings in service conditions. For continuous steel trusses, however, it is advisable to determine these partial span loadings and their effects in order to obtain maximum chord stresses and provide for possible reversal of stress in the panels near the center piers. Consequently, for the maximum moment curves, illustrated in Fig. 21, only full span loads have been considered. Note how one span at a time is loaded and the results combined to give the final maximum moments for dead load plus live load.

Curves of maximum shear are not given in this discussion, although typical influence lines for shear at various points are shown in Fig. 20 (*e*) and (*f*). To obtain the maximum shear at any point within the span, a part-span load must be used. To get its length it is not necessary to compute any values for any part of the influence line, however, as in the case of moments. This is because the load divide point coincides with the point at which the shear is wanted or is in the same panel if a through girder is being analyzed. For reinforced concrete beams of constant section, there is no practical need to find the maximum shears at any interior point, except the center of the span. From this value and the maximum at the support, stirrup spacing may be determined closely enough. If the beams are haunched, the maximum shear at the quarter-point, or at the point of the haunch, should be found in order to keep the unit shear over the reduced section within specification limits.

Case I.—Four 20-ft. spans are used. (See Table 2.) Each span has the same cross-section and, consequently, the $\frac{I}{L}$ ratio is the same for all. As the ends are freely supported, however, the $\frac{I}{L}$ ratio for the end spans has been decreased by one-fourth, the stiffness of a span with pinned ends being only three-fourths as great as when the ends are fixed.* The numbers in circles indicate the relative stiffness of each span. Units are in thousands of foot-pounds.

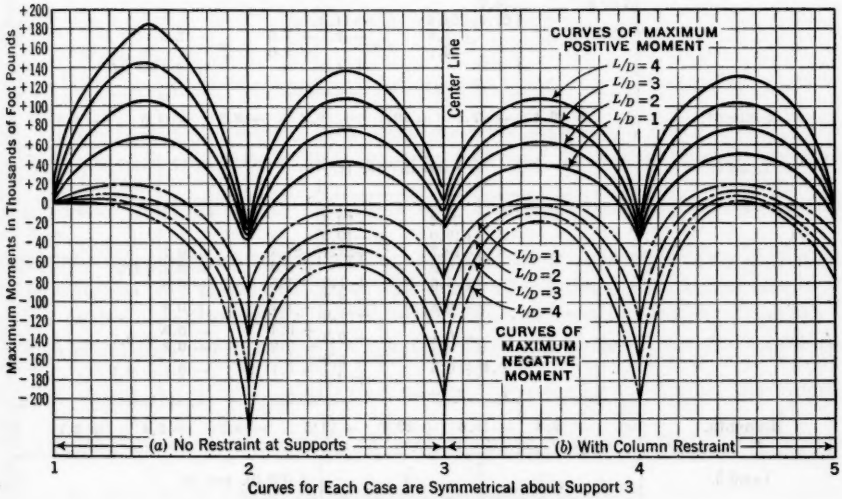
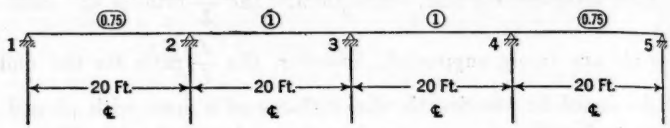


FIG. 21.—CURVES OF MAXIMUM MOMENT.

Moments are found at all supports and in the center of each span, due to a uniform load of 1 000 lb. per ft. over one span at a time. These moments are then combined to give maximum positive and maximum negative moments at each point, due to live load. These two values for each point are then added algebraically to give the moments due to a dead load of 1 000 lb. per ft. over the entire four spans of the continuous system. Live load moments are then combined with dead load moments to obtain the ordinates for the curves of total maximum positive and total maximum negative moments for a ratio, $\frac{L}{D}$, equal to 1. To get curves of moments for any other ratio of $\frac{L}{D}$, the live load moments are first multiplied by the necessary ratio before combining them with those of the dead load. These curves (Fig. 21 (*a*)), are quite easily plotted with no other points than those given, inasmuch as the middle three-fifths (approximately) for each span is a parabola. Curves are shown for $\frac{L}{D}$ ratios of 1, 2, 3, and 4.

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 925.

TABLE 2.—COMPUTATION OF MAXIMUM MOMENTS IN A FOUR-SPAN BEAM.



Load 1.	1 000 lb. per ft.							
	−33.3	−33.3						
	+33.3	−16.7						
		+21.4	−28.6	+14.3				
			+ 3.6	− 7.1	+ 7.2	− 3.6		
			− 2.1		− 1.0			
		+ 1.5		+ 1.0		+ 2.1	− 1.5	
Moments.	0	+ 36.4	− 27.1	− 10.4	+ 6.2	+ 2.3	− 1.5	− 0.8	0
Load 2.			1 000 lb. per ft.					
			−33.3	−33.3				
			+19.0	−19.5				
			−10.7	+21.4	−21.4	+10.7		
			− 0.8		+ 3.0	− 6.1	+4.6	0
			+ 6.6	+ 1.5	− 1.5	− 0.8		
			− 0.8	− 3.3	− 1.7	+ 0.8		
				+ 1.6				
			− 0.1	+ 0.2	+ 0.4	− 0.9		
					− 0.2	+ 0.2	+ 0.7	
		− 0.4	+ 0.5	− 0.2	− 0.1	− 0.1	+ 0.1	
				+ 0.1				
Moments.	0	− 9.8	− 19.6	+ 29.4	− 21.5	− 8.1	+ 5.4	+ 2.7	0
Load 3.					1 000 lb. per ft.			
Moments.	0	+ 2.7	+ 5.4	− 8.1	− 21.5	+29.4	−19.6	− 9.8	0
Load 4.								1 000 lb. per ft.
Moments.	0	− 0.8	− 1.5	+ 2.3	+ 6.2	−10.4	−27.1	+36.4	0
$+M_{L1}$	0	+ 39.1	− 5.4	+ 31.7	+ 12.4			
$-M_{L1}$	0	− 10.6	− 48.2	− 18.5	− 43.0			
M_D	0	+ 28.5	− 42.8	+ 13.2	− 30.6			
$+M_{D+L1}$	0	+ 67.6	− 37.4	+ 44.9	− 18.2			
$-M_{D+L1}$	0	+ 17.9	− 91.0	− 5.3	− 73.6			
$+M_{L2}$	0	+ 78.2	+ 10.8	+ 63.4	+ 24.8			
$-M_{L2}$	0	− 21.2	− 96.4	− 37.0	− 84.0			
								Symmetrical about center line.
$+M_{D+L2}$	0	+106.7	− 32.0	+ 76.6	− 5.8			
$-M_{D+L2}$	0	− 7.3	−139.2	− 23.8	−114.0			

TABLE 2.—(*Continued.*)

$+M_{L3}$	0	+117.8	+ 16.2	+ 95.1	+ 37.2
$-M_{L3}$	0	− 31.8	−144.6	− 55.5	−129.0
Symmetrical about center line.					
$+M_{D+L3}$	0	+145.8	− 24.6	+108.8	+ 6.6
$-M_{D+L3}$	0	− 3.3	−187.4	− 42.3	−159.6
$+M_{L4}$	0	+156.4	+ 21.6	+126.8	+ 49.6
$-M_{L4}$	0	− 42.4	−192.8	− 74.0	−172.0
Symmetrical about center line.					
$+M_{D+L4}$	0	+184.9	− 21.2	+140.0	− 19.0
$-M_{D+L4}$	0	− 13.9	−253.6	− 60.8	−202.0

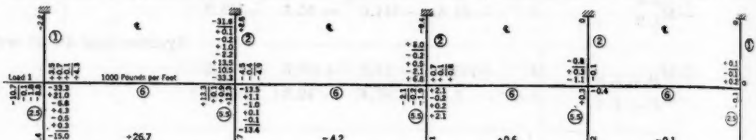
Case II.—A three-story reinforced concrete warehouse, 80 by 160 ft. by 42 ft. high, with columns on 20-ft. centers each way, is used as the structure for analysis. (See Table 3.) Sections were obtained (designed) for a $\frac{L}{D}$ ratio of 2.5. Moments are computed for a second-story four-span beam restrained by columns both above and below. Moments of inertia, I, were computed, using the entire concrete section, but ignoring the steel reinforcement. A width of slab equal to one-fourth the span length was used as the T for the beams and was considered in the moment of inertia. From the $\frac{I}{L}$ values, the relative stiffness, shown at the middle of each member, was computed. These same values were used for all ratios of $\frac{L}{D}$, because it is easily possible to make a design for each loading without altering the relative stiffness of each member.

To obtain the curves of maximum positive and negative moment (Fig. 21 (*b*)), the same procedure used for Case I is followed. Maximum column moments are not shown in the final summation, but can easily be obtained by combining the values shown.

Case II is a good example of the form of computations recommended by the author. The method of "Moment Distribution"* is admirably adapted to a simple and easily checked arrangement of figures. There is some difference of opinion as to the most desirable sign convention to be used. The writer made a somewhat detailed study of this question, comparing different conventions, and concluded that the one used by the author was to be preferred. It not only harmonizes with the usual practice of defining positive and negative moment in a beam, but, in complicated frames, there seems to be no chance of an error in sign remaining undetected when moments are distributed and the equilibrium of each joint is checked. The important factor, however, is that the method is just as workable, regardless of what sign convention is dictated by personal preference.

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 920.

TABLE 3.—COMPUTATION OF MAXIMUM MOMENTS IN A THREE-STORY REINFORCED CONCRETE WAREHOUSE.

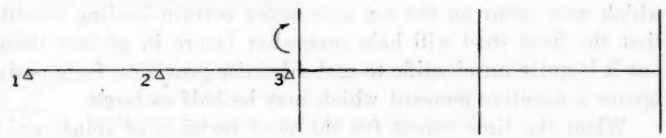


Load 3.							1 000 lb. per ft.		
Moments.	− 0.3	+ 0.7	+ 1.7	+ 4.2	− 3.6	− 11.5	−27.1	+23.1	−26.7	−10.7	− 4.2	+ 2.3
Load 4.											1 000 lb. per ft.	
Moments.	+ 0.1	− 0.1	− 0.4	− 0.8	+ 0.6	+ 2.1	+ 5.0	− 4.2	−13.4	−31.6	+26.7	−15.0
$+M_{L1}$	+ 2.4	+ 27.4	+ 1.7	+ 4.2	+ 23.7	+ 7.1						
$-M_{L1}$	−15.3	− 4.3	− 42.7	− 40.9	− 7.8	− 38.6						
M_D	−12.9	+ 23.1	− 41.0	− 36.7	+ 15.9	− 31.5						
$+M_{D+L1}$	−10.5	+ 50.5	− 39.3	− 32.5	+ 39.6	− 24.4	Symmetrical about center line.					
$-M_{D+L1}$	−28.2	+ 18.8	− 83.7	− 77.6	+ 8.1	− 70.1						
$+M_{L2}$	+ 4.8	+ 54.8	+ 3.4	+ 8.4	+ 47.4	+ 14.2						
$-M_{L2}$	−30.6	− 8.6	− 85.4	− 81.8	− 15.6	− 77.2						
$+M_{D+L2}$	− 8.1	+ 77.9	− 37.6	− 28.3	+ 63.3	− 17.3	Symmetrical about center line.					
$-M_{D+L2}$	−43.5	+ 14.5	−126.4	−118.5	+ 0.3	−108.7						
$+M_{L3}$	+ 7.2	+ 82.2	+ 5.1	+ 12.6	+ 71.1	+ 21.3	Symmetrical about center line.					
$-M_{L3}$	−45.9	− 12.9	−128.1	−122.7	− 23.4	−115.8						
$+M_{D+L3}$	− 5.7	+105.3	− 35.9	− 24.1	+ 87.0	− 10.2						
$-M_{D+L3}$	−58.8	+ 10.2	−169.1	−159.4	− 7.5	−147.3						
$+M_{L4}$	+ 9.6	+109.6	+ 6.8	+ 16.8	+ 94.8	+ 28.4	Symmetrical about center line.					
$-M_{L4}$	−61.2	− 17.2	−170.8	−163.6	− 31.2	−154.4						
$+M_{D+L4}$	− 3.3	+132.7	− 34.2	− 19.9	+110.7	− 3.1						
$-M_{D+L4}$	−74.1	+ 5.9	−211.8	−200.3	− 15.3	−185.9						

Results of Computations.—This discussion is submitted to show primarily the application of the author's method of analysis to a specific problem.* It would not be complete, however, unless some pertinent conclusions about the particular problem investigated were included.

The correct values for the maximum moments at critical points are compared with the values obtained, using the conventional moment coefficients found in the design codes for reinforced concrete. (See Table 4.) The coefficients in Article 107 of the report of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete (1924)† are used for Table 4 (*a*) and the coefficients in Article 110 (*a*) and (*b*) are used for Case II, Table 4 (*b*). The length of span used in Table 4 (*a*) for code values is the full length, that is, 20 ft. In Table 4 (*b*), in which columns restrain the beams, code values are computed for a clear span of 18.5 ft. Therefore, in order to make a fair comparison, the correct moments at the face of the columns have been computed and tabulated. That explains the difference in values at the supports shown in Table 4 (*b*) and in Case II. The computations, Case II, are for the center line of the support.

TABLE 4.—COMPARISON OF CORRECT MAXIMUM MOMENTS WITH THOSE DETERMINED BY THE USE OF CONVENTIONAL MOMENT COEFFICIENTS.



(*a*) No restraint. (*b*) Column restraint.

$\frac{L}{D}$ =		(*a*)				(*b*)				
1	Correct *M*.....	0	+ 67.6	− 91.0	+ 44.9	− 73.6	− 15.4	+ 50.5	− 67.1	− 62.2	+ 39.6	− 55.3
	Code *M*.....	0	+ 80.0	− 80.0	+ 66.6	− 66.6	− 57.0	+ 57.0	− 57.0		+ 42.8	− 57.0
	Error, percentage.	0	+ 18.3	− 12.1	+ 32.5	− 9.5	+270.0	+ 12.9	− 15.0	− 8.4	+ 8.1	+ 3.1
2	Correct *M*.....	0	+106.7	−139.2	+ 76.6	−114.0	− 24.0	+ 77.9	−101.5	− 95.1	+ 63.3	− 86.1
	Code *M*.....	0	+120.0	−120.0	+100.0	−100.0	− 85.5	+ 85.5	− 85.5		+ 64.2	− 85.5
	Error, percentage.	0	+ 12.5	− 13.8	+ 30.6	− 12.3	+257.0	+ 9.8	− 15.8	− 10.1	+ 1.4	− 0.7
3	Correct *M*.....	0	+145.8	−187.4	+108.3	−159.6	− 32.6	+105.3	−135.9	−128.9	+ 87.0	−116.9
	Code *M*.....	0	+160.0	−160.0	+133.3	−133.3	−114.0	+114.0	− 114.0		+ 85.6	−114.0
	Error, percentage.	0	+ 9.7	− 14.6	+ 23.1	− 16.5	+249.0	+ 8.3	− 16.1	− 11.0	− 1.6	− 2.5
4	Correct *M*.....	0	+184.9	−235.6	+140.0	−202.6	− 41.2	+ 32.7	−170.3	−160.9	+110.7	−147.7
	Code *M*.....	0	+200.0	−200.0	+166.6	−166.6	−142.5	+142.5	−142.5		+107.0	−142.5
	Error, percentage.	0	+ 8.2	− 15.1	+ 19.0	− 17.8	+246.0	+ 7.4	− 16.5	− 11.4	− 3.3	− 3.5

Figures showing the percentage of error in code moments should interest many designers. For both unrestrained and restrained beams, code coefficients give values too high for positive moment in the centers of the spans and values

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 927.

† *Loc. cit.*, October, 1924, Papers and Discussions, p. 1153 *et seq.*

too low for negative moment at the supports. The difference is enough to warrant a revision in all the coefficients applied to continuous systems with little or no restraint, such as that of Case I. In systems like that of Case II, coefficients for the interior spans are sufficiently accurate for practical purposes. The coefficients for the end spans and particularly the end supports are decidedly in error, however, and should be revised.

The design codes make no reference whatever to the existence of negative moment at the center of spans. (See Fig. 21.) Even for a $\frac{L}{D}$ ratio as small as unity, negative moment may occur over the full length of interior spans subject to little end restraint. Consequently, reinforcing steel is needed at the top of the beam over its full length. Note the values for a $\frac{L}{D}$ ratio of 4. The maximum negative moment at the center of the second span is 60.8. The maximum positive moment at the same point is 140.0. Most designers will place steel in the bottom but not in the top, disregarding the fact that the top needs 43% of the total used in the bottom. Even for spans subject to column restraint, the negative moment is large enough over the greater part of the interior spans to justify a decided change in current design practice. Some will argue that there is no need for worrying about this because the floor steel in the slab will assist the beam in providing resistance to the tension which may occur on the top side under certain loading conditions. It is true that the floor steel will help somewhat (more in girders than in the beams), but it is quite unscientific to make definite provision for a positive moment and ignore a negative moment which may be half as large.

When the time comes for the next revision of reinforced concrete design codes, the writer hopes that the responsible authorities will omit all values for moment coefficients, thus placing the function of analysis entirely upon the shoulders of the designer. This will not only encourage the engineer to develop more self-reliance and initiative, but it will pay deserved premiums to the man with the technical ability to design structures intelligently. It must be remembered that too many designers will accept a "half-complete" code as the authority for a complete design. "Their's not to question why * * *". However, if the responsible authorities still insist that there must be more rules for the game, then it is advisable to define explicitly those limited cases to which the rules apply. It will impose some check on the designer who is inclined to take too much latitude.

With the author's method, maximum moments and shears for any design subject to any condition of uniform or concentrated loads are so readily computed that there is little need for using arbitrary coefficients in any case. Although the preceding discussion revolves chiefly around equal span systems subject to uniform loads, systems with equal or unequal spans subject to concentrated loads are just as easily treated.

Frames Subject to Lateral Displacement.—The writer also wishes to illustrate the facility of applying the author's method to a problem usually considered quite complex, that is, that of a mill-building bent, with a stepped column subject to a lateral thrust from a traveling crane. The problem is

complicated by the variation in moment of inertia of the column due to the different sections used above and below the crane girder seat. (See Fig. 22.) The moments in the columns are wanted, due to the effect of a lateral concentrated load of 6 000 lb. on each column; this is the horizontal thrust of the crane, acting 27 ft. above the base. The moments are first found for the horizontal load acting on one column. Since there is a similar load acting on the other column, the final moments are obtained by combining those due to a single load on opposite hand bents. Vertical loads and wind loads are ignored in this illustration. Moments due to such loads would be computed in a similar manner. For a complete design, they would naturally have to be considered.

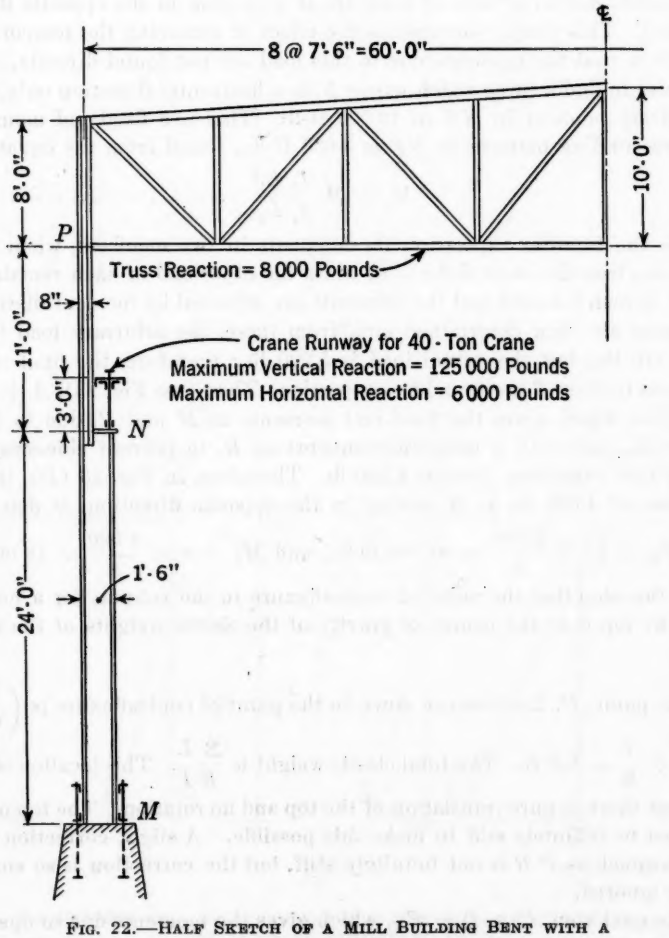


FIG. 22.—HALF SKETCH OF A MILL BUILDING BENT WITH A STEPPED COLUMN.

The base of the column is considered fixed. The ratio of the stiffness of the truss to the stiffness of the column is more than 10. This makes the upper

end of the column practically fixed, so that in the following analysis the $\frac{I}{L}$ value for the truss is taken as infinity. The final moments are consequently slightly in error. If the exact moments are wanted, they may be found by using the correct stiffness factor for the truss and releasing Joints P and R (Fig. 23) after the fixed-end moments are found. The correction, however, is too small to have practical significance.

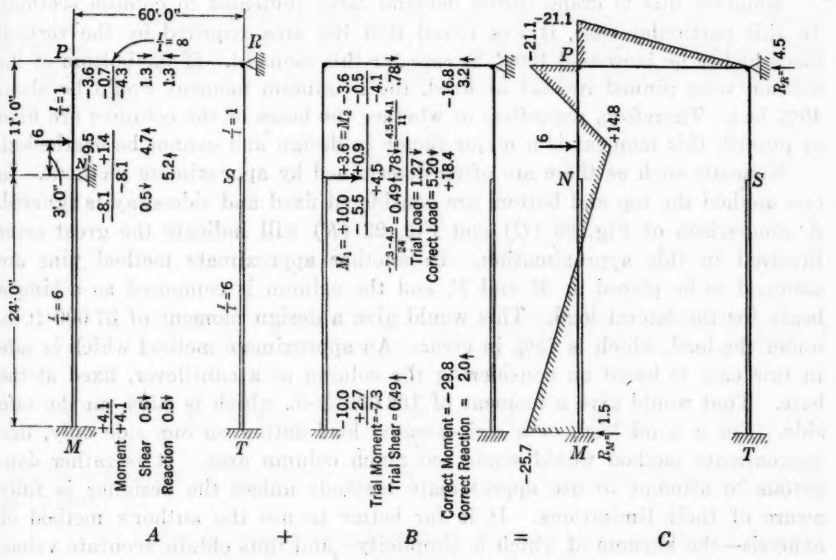
In the first operation, Fig. 23 (A), Column MNP is temporarily supported at N, where the change in column section occurs, and the moments on the resulting two-span continuous beam are found due to the load of 6 000 lb. Reactions at M, N, and P are found; that at N is 5 200 lb. In Fig. 23 (B), the moments due to a force of 5 200 lb. at N, acting in the opposite direction, are found. This simply determines the effect of removing the temporary support. Note that the moments due to this load are not found directly, however. It is easier to find a force which moves N in a horizontal direction only, causing an arbitrary moment in MN of 10 000 ft-lb. This is a fixed-end moment and the corresponding moment in NP is 3 630 ft-lb., found from the equation,

$$M_2 = M_1 \frac{I_2 L_1^2}{I_1 L_2^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

which is the familiar expression for moments in two members, when one end of each is given the same deflection, while the other end of each remains fixed. Joint N is then released and the moments are adjusted by moment distribution. The shears are then determined and, from these, the arbitrary load is found to be 1 270 lb.; but the actual load is 5 200 lb.; therefore, the final moments and shears in Step B are found by proportion. Then (see Fig. 23), $A + B = C$, a condition which gives the fixed-end moments at M and P, due to the load of 6 000 lb., and with a temporary support at R, to prevent side-sway. The value of this temporary force is 4 500 lb. Therefore, in Fig. 23 (D), the effect of a force of 4 500 lb. at R, acting in the opposite direction, is determined. Thus, $M_M = 27 \times \frac{4\,500}{2} = 60\,800$ ft-lb., and $M_P = 8 \times \frac{4\,500}{2} = 18\,000$ ft-lb. Note in this step that the point of contraflexure in the column for a horizontal load at the top is at the center of gravity of the elastic weights of the column.

From the point, P, the distance down to the point of contraflexure is $\left(\frac{1}{6} \times 23 + 5.5\right) \div \frac{7}{6} = 8.0$ ft. The total elastic weight is $\frac{\Sigma L}{E I}$. This location is correct only when there is pure translation of the top and no rotation. The top member, $P R$, must be infinitely stiff to make this possible. A slight correction may be made inasmuch as $P R$ is not infinitely stiff, but the correction is so small that it can be ignored.

In the next step, $C + D = E_R$, which gives the moments due to one of the 6 000-lb. loads. To account for the other load of 6 000 lb. on the other column, the moments shown in E_R are added to opposite-hand moments to obtain the final values in Fig. 23(F).



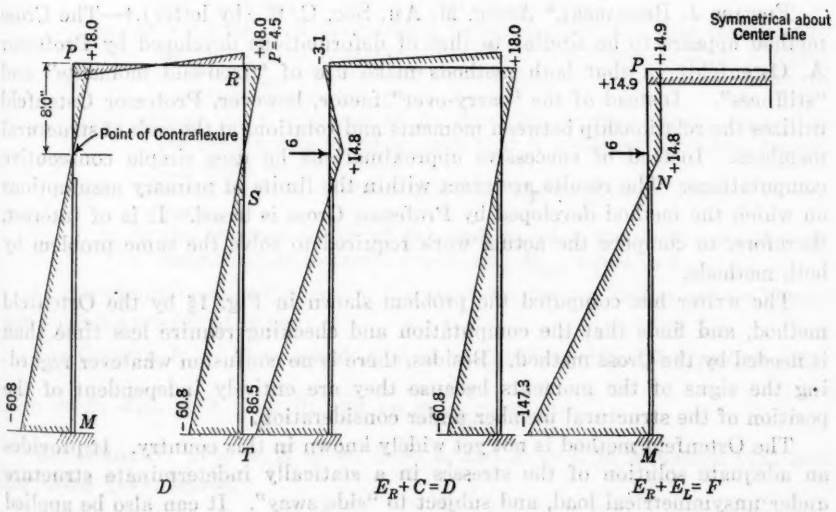


FIG. 23.—MOMENTS IN A MILL BUILDING BENT DUE TO LATERAL CRANE THRUST, IN THOUSANDS OF FOOT-POUND UNITS.

This problem not only illustrates the application of the Cross method to frames subject to side-sway, but, at the same time, points out some interesting conclusions about mill-building bents of this type.

Moments due to crane thrust demand large increases in column sections. In this particular case, it was found that the area required by the vertical loads had to be increased 100% to care for this moment. If the bottom of the column were pinned instead of fixed, the maximum moment would be about 40% less. Therefore, regardless of whether the bases of the columns are fixed or pinned, this moment is a major factor in design and cannot be overlooked.

Moments such as these are often determined by approximate methods. In one method the top and bottom are considered fixed and side-sway is ignored. A comparison of Fig. 23 (*C*) and Fig. 23 (*F*) will indicate the great error involved in this approximation. In another approximate method pins are assumed to be placed at *M* and *P*, and the column is computed as a simple beam for the lateral load. This would give a design moment of 37 000 ft-lb. under the load, which is 75% in error. An approximate method which is safe in this case is based on considering the column as a cantilever, fixed at the base. That would give a moment of 162 000 ft-lb. which is 9.5% on the safe side. For a wind load, or a concentrated load acting on one side only, this approximate method would waste too much column area. It is rather dangerous to attempt to use approximate methods unless the designer is fully aware of their limitations. It is far better to use the author's method of analysis—the keynote of which is simplicity—and thus obtain accurate values for design purposes.

Edward J. Bednarski,* Assoc. M. Am. Soc. C. E. (by letter).†—The Cross method appears to be similar to that of deformations developed by Professor A. Ostenfeld‡ in that both methods make use of "fixed-end moments" and "stiffness". Instead of the "carry-over" factor, however, Professor Ostenfeld utilizes the relationship between moments and rotations at the ends of structural members. Instead of successive approximations he uses simple consecutive computations. The results are exact within the limits of primary assumptions on which the method developed by Professor Cross is based. It is of interest, therefore, to compare the actual work required to solve the same problem by both methods.

The writer has computed the problem shown in Fig. 1§ by the Ostenfeld method, and finds that the computation and checking require less time than is needed by the Cross method. Besides, there is no confusion whatever regarding the signs of the moments because they are entirely independent of the position of the structural member under consideration.

The Ostenfeld method is not yet widely known in this country. It provides an adequate solution of the stresses in a statically indeterminate structure under unsymmetrical load, and subject to "side sway". It can also be applied to continuous beams on tall columns as well as to multi-storied bents and

* Civ. Engr., Sacramento, Calif.

† Received by the Secretary, September 12, 1930.

‡ "Deformations Methode," by A. Ostenfeld, Copenhagen, 1926.

§ *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 922.

multiple arches on high piers. The computation required by this method is outlined herein, as applied to the problem under discussion.

The stiffness factors, μ, and the fixed-ended moments, M^0, to be used in the present demonstration are the same as those assumed by Professor Cross (see Fig. 1). The signs of the moments are considered positive in the clockwise direction. To solve the problem by the Ostenfeld method, first determine values of the rotation factors, ζ_{oa}, at the ends of each structural member, by means of the formulas:

$$\zeta_{oa} = \frac{Z_{oa}}{Z_{aa}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

and,

$$\tau_a = \frac{1}{Z_{aa}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

in which,

$$Z_{oa} = -\left[M^0_{am} + \frac{2\,\mu_{am}}{1 + 4\,\mu_{am}\,\tau_m}(\zeta_{om} - \tau_m\,M^0_{ma})\right] - M^0_{ai} - M_{ak} \ldots (12)$$

and,

$$Z_{aa} = 4\,\mu_{am}\,\frac{1 + 3\,\mu_{am}\,\tau'_m}{1 + 4\,\mu_{am}\,\tau_m} + 4\,\mu_{ai} + 3\,\mu_{ak} \ldots\ldots\ldots\ldots (13)$$

The subscripts in Equations (10) to (13), inclusive, may be defined briefly as follows:*

$a\,i$ refers to one or more members, $a\,i$, that are connected at Joint a; the other end, i, of the members is fixed.

$a\,k$ refers to one or more members, $a\,k$, that are connected to Joint a; the other end, k, of the members is fixed.

$o\,a$, as a subscript of the rotation factor, ζ, identifies it as referring to the end, a, of a member at which the moment, M_{am}, is equal to zero; Z_{oa} is the moment necessary at the end, a, of a member, to produce the rotation, ζ_{oa}.

$a\,a$ appears as a subscript to Z; thus, Z_{aa} is the moment applied at the end, a, of a member, such that it produces a rotation factor, ζ_{aa}, equal to —1.

$a\,m$, as a subscript of M, identifies it as the moment at Point a in any beam.

τ_m is the rotation factor at the end, m, of a member, $a\,m$, of such magnitude as to create a moment, M_{am}, equal to 1.

In the example under discussion, the necessary factors for the various beams and columns are found to be:

For Column $B\,A$: $B = a$ and $A = m$ in Equation (12) and Equation (13); $Z_{OB} = 0$; $Z_{BB} = 8$; $\zeta = 0$; and $\tau_B = \frac{1}{Z_{BB}} = \frac{1}{8}$.

For Joint C: $C = a$, $B = m$, $G = k$, and $F = i$, in Equation (12) and Equation (13); hence,

$$Z_{OC} = -100 + \frac{2 \times 4}{1 + 4 \times 4 \times \frac{1}{8}} \times \frac{1}{8} \times 100 - 50 - 110 = -\frac{880}{3}$$

* See also "Deformations Methode," by A. Ostenfeld, Copenhagen, 1926.

$$Z_{CC} = 4 \times 4 \times \frac{1 + 3 \times 4 \times \frac{1}{2}}{1 + 4 \times 4 \times \frac{1}{8}} + 4 \times 1 + 3 \times 2 = \frac{70}{3}$$

$$\zeta_{CO} = \frac{88}{7}$$

and,

$$\tau_C = \frac{1}{Z_{CC}} = \frac{3}{70}$$

For Joint D: $D = a$ and $C = m$ in Equation (12) and Equation (13); $Z_{OD} = -\frac{1020}{13}$; $Z_{DD} = \frac{230}{13}$; $\zeta_{OD} = -\frac{102}{23}$; and $\tau_D = \frac{13}{230}$.

For Joint E: $E = a$ and $D = m$ in Equation (12) and Equation (13); $Z_{OE} = \frac{3060}{193}$; $Z_{EE} = \frac{2082}{193}$; $\zeta_{OE} = \frac{510}{347}$; and $\tau_E = \frac{193}{2082}$.

The actual rotation factors, ζ_m, are finally determined for each joint in succession by the formula:

$$\zeta_m = \frac{1}{1 + 4\,\mu_{am}\,\tau_m}\,(\zeta_{om} - 2\,\mu_{am}\,\tau_m\,\zeta_a - \tau_m\,M^0{}_{ma}) \quad \ldots\ldots\ldots (14)$$

The actual values for the problem in Fig. 1 are thus found to be: $\zeta_E = \frac{2495}{1041}$; $\zeta_D = -\frac{1085}{347}$; $\zeta_C = -\frac{497}{347}$; $\zeta_B = \frac{3228}{694}$; $\zeta_P = \frac{1}{2}\,\zeta_C = \frac{497}{694}$; and $\zeta_A = \zeta_B = 0$.

The final moments are determined by means of the equations:

$$M_{am} = 2\,\mu_{am}\,(2\,\zeta_a + \zeta_m) + M^0{}_{am} \quad \ldots\ldots\ldots\ldots\ldots (15)$$

and,

$$M_{ma} = 2\,\mu_{am}\,(2\,\zeta_m + \zeta_a) + M^0{}_{ma} \quad \ldots\ldots\ldots\ldots\ldots (16)$$

in which, M^0, denotes the original fixed-ended moment at the end of the beam indicated. The values of M, thus determined are, respectively,

$$M_{AB} = 18\,\frac{200}{347} = 18.576$$

$$M_{BA} = 2\,M_{AB} = 37.152$$

$$M_{BC} \text{ (computed independently)} = -\,M_{BA} = -\,37.152$$

$$M_{CB} = 114\,\frac{82}{347} = 114.236$$

$$M_{CF} = 101\,\frac{141}{347} = 101.406$$

$$M_{CD} = -\,259\,\frac{317}{347} = -\,259.913$$

$$M_{CG} = 44\,\frac{94}{347} = 44.271$$

$$M_{DC} = 23\,\frac{49}{347} = 23.141$$

$$M_{DE} \text{ (computed independently as a check)} = -M_{DC} = -23.141$$

$$M_{ED} = \frac{3470}{347} = 10$$

and,

$$M_F = -M_{ED} = -10$$

It should be noted as a check that the sum of all moments around Point C equals zero.

S. N. MITRA,* JUN. AM. SOC. C. E. (by letter).†—Despite some limitations the method described in this paper is quite unique and satisfactory. In the writer's opinion, however, it has the decided disadvantage that it is not conveniently reducible to a general equation that can be used when the principles themselves have been mastered.

Furthermore, the writer believes that, for less complicated problems than the one the author has illustrated, such as those which occur in practice, other methods of analysis, as, for example, slope deflection and the graphical method of conjugate points, will be found more accurate, and quicker and easier to use.

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† Received by the Secretary, July 25, 1930.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

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STRESSES IN GRAVITY DAMS BY PRINCIPLE OF LEAST WORK

Discussion*

By Messrs. William Gore, L. J. Mensch, Johannes Skytte, and Lars R. Jorgensen.

William Gore,† M. Am. Soc. C. E. (by letter).‡—Since the English tests of nearly a quarter of a century ago were published§ the general problem of the stresses in gravity dams has not received much attention. At that time this problem appeared to have been solved, but Mr. Jakobsen now raises it again from a new angle. The English tests indicated that, except at or near the foundations of a dam, the classical assumption of the trapezium or straight-line law, although not correct, was not so far wrong as to cause any anxiety concerning the safety of dams designed in accordance therewith, and all subsequent designs have followed these assumptions.

This situation has been a source of gratification to Mr. J. S. Wilson and the writer, who conducted most of the English tests. These tests had a direct bearing on the design of the first raising of the Assuan Dam, and their accuracy is a matter of considerable moment in discussing the author's paper. Mr. Jakobsen draws special attention to this, but he states‖ that "the tests are not wholly satisfactory on account of the possibility of considerable local strains where the weights are attached, and the uncertainty as to the elastic behavior of the rubber, the time effect, etc." All tests, of course, are subject to possible inaccuracies such as those indicated by the author. They are also subject to inaccuracies due to the fact that the general assumptions and arrangements of the experimenters may not have been in agreement with the practical case they are intended to represent; but in the case of the English tests every effort was made to keep such inaccuracies as small as possible.

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* Discussion of the paper by B. F. Jakobsen, M. Am. Soc. C. E., continued from September, 1930, *Proceedings*.

† Cons. Civ. Engr. (Gore, Nasmith & Storrie), Toronto, Ont., Canada.

‡ Received by the Secretary, September 22, 1930.

§ *Minutes of Proceedings*, Inst. C. E., Vol. CLXXII, Session 1907–08, Pt. II, Lond., 1908.

‖ *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, pp. 1629–1630.

The work on gravity dams extended over a period of about four years and involved a considerable amount of labor, most of which was not seen in the published results. Everything possible was done to make the results classical. The exaggerated virtual weights of 6 000 lb. per cu. ft. for the masonry, and 2 500 lb. per cu. ft. for the water, were adopted in order that the strains produced could be large enough to be measured with reasonable accuracy while maintaining the same relative densities of concrete to water. The dam sections unloaded were distorted in form and it was only when fully loaded that the correct profiles were secured.

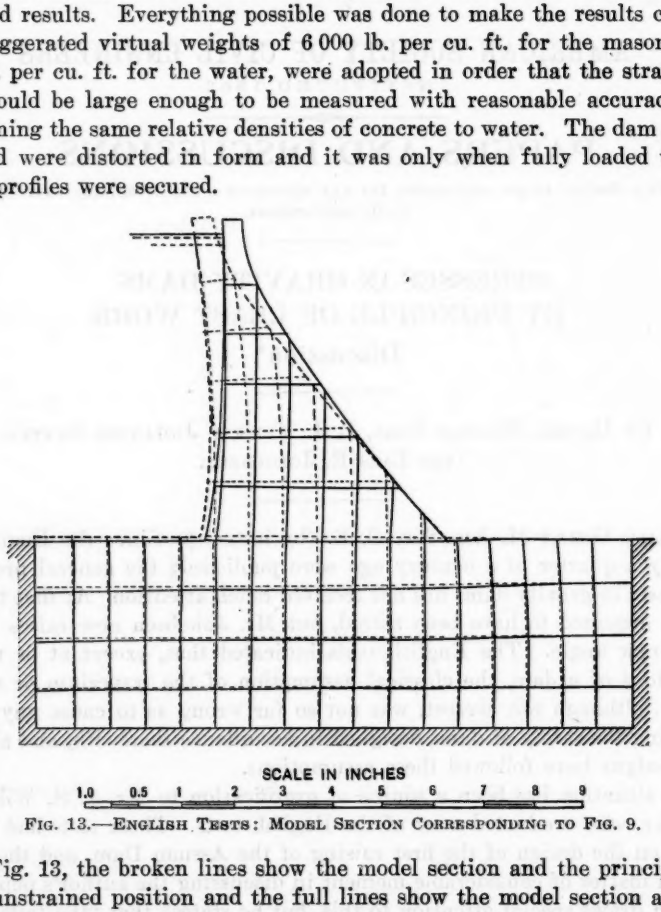

FIG. 13.—ENGLISH TESTS: MODEL SECTION CORRESPONDING TO FIG. 9.

In Fig. 13, the broken lines show the model section and the principal lines in the unstrained position and the full lines show the model section and principal lines when fully strained with both "gravity" and "water weights". Test samples, cut from the same slabs as the sections, similarly treated and prepared, were subjected to various loadings under the same conditions as the sections, and were found to give an approximately straight-line relation between stress and strain within the limits of the tests.* Owing to the time effect of stress upon rubber the models and similarly treated rubber sample tests were loaded for 24 hours, then rested for 48 hours, and the cycle repeated again and again until a permanent régime had been established. The measurement photographs of the loaded sections were taken after the loads had been applied 24 hours, and the unloaded sections, 48 hours after the loads had been taken off. Some time after loading, the pads conveying the water pressures were gently rolled sidewise to allow the soft rubber under them to creep to freedom. The models

* *Minutes of Proceedings*, Inst. C. E., Vol. CLXXII, Session 1907–08, Pt. II, Lond., 1908.

and fixed camera were in a room in which none beside the experimenters was allowed to enter, and great care was taken that the precise distance relationship between the camera and object was undisturbed.

The later measurements were made upon large photographic negatives, especially prepared on plate glass, from 1½ to 2 times the actual size, by means of especially designed micrometer gauges. The gauge lines on the glass were opaque; they were projected by a lens in a vertical optical bench on the negatives and adjusted with a low-power microscope so that the black images of the gauge lines of the micrometer were caused to register upon the transparent lines or circles on the negatives. Length measurements to 0.001 in. were measured accurately and lengths of 0.0001 in. could be estimated in most cases. Angles of shear were read in the same manner accurately to tangents of 0.002.* All principal measurements were taken along lines midway between the load points, and the effects of the concentration of the loads are hardly discernible as the published measurements show. This is supported by theoretical considerations. In interpreting the stresses from the strains the calibration of every through line was adopted.

It is possible that the elastic constants of a material such as rubber, even of the very best quality such as that adopted, do vary from point to point to some extent, but observations made during the English tests did not show these variations to be of great importance. The two sections (Figs. 8† and 9†) constructed at different times compare very favorably; but in any practical case—whether by this or any other method—it is desirable that two or more models exactly alike be used and the results averaged. The results of the English tests have a high degree of accuracy and may reasonably be compared with the author's equations for n_y, n_x, and t.

At the time the English tests were made, it was held that a correct mathematical solution was not possible owing to the unknown and possibly arbitrary conditions at the base of the dam. The French engineer, Maurice Lévy, had attempted the solution by means of differential equations, but these did not take into account the base conditions. For a triangular dam, Lévy's solution was in accordance with the classical assumption, but in the case of a vertical wall the stresses showed considerable departures from straight lines; tensions were not indicated in either case. The first of the English tests were with a triangular dam of minimum section with a small base resting upon steel balls and free to travel horizontally.‡ It was only prevented from doing so by the application along the base of a system of forces equal and opposite to the water pressure and representing the shearing forces. These could be varied in any manner so long as static equilibrium was maintained.

When the applied distribution of shear at the base corresponded with the classical assumption no tensions were observed in the dam, although the line of resistance was theoretically following the down-stream boundary of the middle third. The curves of shear on horizontal lines at all levels were similar; they were nearly a straight line slightly turned up at the up-stream end and

* *Engineering*, May 22, 1908.

† *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1630.

‡ *Engineering*, August 4, 1905.

down at the down-stream end. The curves for n_y were similar. When a test was made with the classical shear distribution reversed, the effect of the change on the distribution of stress in the section was not considerable for more than 40% up from the base of the section, but the changes below the 40% level were extensive. Local tensions were observed at the up-stream heel. These tests established the type of stress curves in triangular dams not too near the vase, and all subsequent tests of whatever source appear to confirm it.

The sections, Figs. 8 and 9, were developed later to deal in a natural and practical way with the lower part of the dam and the bed-rock foundations in the vicinity, including a considerable portion under the reservoir. For this purpose it was assumed that the rock and masonry were homogeneous and isotropic, and that they had the same elastic constants. The measuring photographic negatives did not include the higher parts of the sections although these were treated at a later date and disclosed n_y-curves similar to Line 3-3, Fig. 9. These tests did not indicate tensions in the upper parts of the dam, and Mr. Wilson and the writer were led to the conclusion that the tensions at the up-stream heel observed were due to the influence of the restricted connection to the great mass of the rock under the bed of the reservoir and the water pressures upon it. The tensions observed were quite local. These general results, although of considerable importance in dam design, except above the 40% level, appear to be beyond the problem dealt with by Mr. Jakobsen, which relates to a simple triangular section in contact with a rigid base at infinity.

In interpreting his results, Mr. Jakobsen has introduced in addition to the conditions of static equilibrium, the requirement that the internal work done by the applied water and the masonry loadings shall be a minimum and this condition replaces the assumption of straight-line stress. The work has involved considerable analysis and is a very valuable contribution to this most important problem. Professor Cain* has checked the author's work, and it may be regarded as accurate within the assumptions adopted.

An examination of the method, in conjunction with the English tests, indicates that the results should apply to the upper parts of a dam, not too near the rectangular top. The results, however, appear more nearly to fit the base conditions only as shown by the English tests, but the dropping of the n_y-curve at the up-stream heel is not nearly so local. This result is to be expected owing to the restrictions at the bases which, although local in the models, appear to be carried up the entire structure in Mr. Jakobsen's calculations. Thus, it must be admitted that there are serious differences between the author's analysis and the English tests, and as Mr. Jakobsen's method is an exceedingly attractive one, it would be interesting to find the cause of the differences—whether they are fundamental or due to some elusive further necessary consideration, or to over-restriction. His close association with the models leads the writer to the presumption that the conditions (both internal and external) applied to them do not nearly correspond with those assumed by the author in his calculations and, therefore, the results must be different.

The most noticeable feature of Mr. Jakobsen's treatment is his assumption of a rigid base and possibly an equivalent at higher levels. In all probability

* *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1639.

this is the cause of the excessive tension indicated and would be very serious indeed, if true, in actual gravity dams. In all the model sections, lines originally horizontal and straight move and tip down stream and also sag, and the change is such as to relieve the possible tension and to reduce maxima of either kind. Fig. 13 shows the strain picture of the English tests traced from photographs corresponding to the author's Fig. 9. The application of a straightedge to the horizontal lines will show the sag mentioned, and it will be noticed that considerable external work must have been done by the section, which somehow must come into any calculation of least work under the model conditions. Thus, the old conclusion is reached, namely, that when the conditions at the base are unknown both as to stress, strain, and work, no calculation is satisfactory, and the assumption of a rigid base or horizontal plane is not much nearer satisfaction than the classical assumption of the straight-line law and may lead to greater errors.

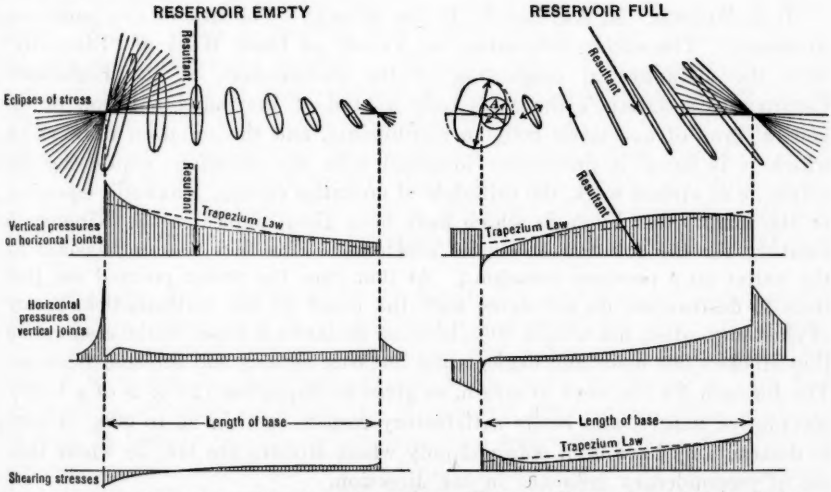

FIG. 14.—STRESSES AT BASE OF MODEL DAM: ENGLISH TESTS, LINE 6–6 OF SAME MODEL AS FIG. 9.

In any actual dam the supports go down into the body of the earth and extraordinary large displacements, including settlements, are only prevented by the unstrained banks at the ends of the dam. Except in very narrow gorges, where gravity sections are now seldom or ever adopted, the elastic bases may be more free than is provided in the models.

The author's reference* to Maupertius is interesting. The application of the principle of least work in this case so far seems to lead in the opposite direction to that indicated by Maupertius. Its application to gravity dams leads not to a beneficient providential situation, but to destructive tensions in structures unable to resist them. It causes them to fail, releasing great bodies of water and spreading death and destruction over extensive territories. In this case, however, it is more providential to know the truth and then meet it than to carry on without the knowledge.

* *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1632.

It seems in all probability that the principle of least work does actually apply to this most interesting problem and that the author has secured a solution for one over-restricted case; but the actual application of it in full to the general problem is beset with difficulties which may prove insuperable. It might be that the author could re-state his problem by assuming the strains at the base of the structure to be proportional to the stresses determined by his present results regarded as a first approximation, and then re-calculate the amount of least work, and so on, successively, until a permanent form had been established. Of course, any assumption that straight lines in the dam before loading remain straight lines after loading is not permissible. Fig. 14 shows some further results from the English tests on the model in Fig. 9; it relates only to the base line, 6-6. It should be noted that the departures from the straight line of the n_y-curves for reservoir empty and reservoir full are approximately reversed.

L. J. MENSCH,* M. AM. SOC. C. E. (by letter).†—The title of this paper is a misnomer. The author confounds the Theory of Least Work of "Elasticity" with the metaphysical conception of the philosophers of the Eighteenth Century. Castigliano's theory is only a kind of shorthand notation of the general laws of an elastic body in equilibrium, and the equation of work on which it is based is practically identical with the equations expressing the principle of virtual work, the principle of potential energy, Maxwell's equation, or the many other methods which have been given names in the Nineteenth Century for the solution of elastic problems. This fact was emphasized by the writer on a previous occasion.‡ At that time the writer pointed out that tests to destruction do not agree with the result of the mathematical theory of elasticity, often not within 50%, because the latter is based on the assumption that Hooke's law holds and neglects the inelastic actions and internal frictions. The formula for the work of strain, as given by Equation (14),§ is of a highly speculative nature, and really satisfactory results (within 25 to 50% of tests to destruction) have been obtained only where stresses are low, or where they are of preponderant influence in one direction.

Consider, for example, the case of a column under concentric axial load. The work performed by the load equals load times shortening of the column and this must be equal to the sum of the work performed by all interior particles, which are assumed to be uniformly loaded and strained, during their shortenings. The modulus of elasticity in compression is derived from this point of view; but according to the mathematical theory of elasticity there are a great many other stresses in such a simple structure. All textbooks in common use show that shearing stresses are acting in such a column in sections inclined to the axis and that these shearing stresses, disregarding internal friction, are a maximum for an inclination of 45°, when they are equal to one-half the axial stresses. The simple method of computing the internal work is incorrect, and the actual work of the axial stresses is only a part of the entire

* Chicago, Ill.

† Received by the Secretary, September 25, 1930.

‡ "On a New Principle in the Theory of Structures," by George F. Swain, Past-President, and Hon. M., Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 654.

§ *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1617.

internal work. No way has been found, since the inception of the mathematical theory of elasticity, to verify experimentally the assumed distribution of work performed on each particle in the various directions of strain. For a proper solution it will probably be necessary to wait until the ultimate constitution of matter has been determined.

The novelty introduced by the author is a non-linear stress distribution in a horizontal section of a dam, the distribution following a curve of the fourth order, given by Equation (15).* He has not tried to show that this distribution of stresses will really produce the least amount of work compatible with the loading conditions; he has only calculated the coefficients of this particular curve to satisfy the hypothetical Equation (14). Other writers have assumed a parabolic stress distribution; and the writer has tried a cubic parabola† to which he was led by comparing the results of tests to destruction on a large number of eccentrically loaded columns.

The writer finds it strange that the author uses the stresses in a horizontal section, which is inclined to the neutral axis of the dam, in such a delicate operation as the solution of Equation (14). From experiments with wedge-shaped beams of cast iron and reinforced concrete the writer knows that the line of failure follows somewhat the line, $AO'B$, in Fig. 15, which for convenience of calculation may be replaced by the dotted line, $A'B'$. An investigation of this section by Mr. Jakobsen with a stress distribution similar to Equation (15) would have shown the point of zero stress a great deal farther down stream than for a horizontal section, namely, at N instead of n in Fig. 15.

The author finds tension at the up-stream face and does not mention that such tension cannot exist in the joints of a non-monolithic structure. Sometimes it is permitted to assume tension in the design of brick masonry, but this is done for the following reason: When brick smokestacks were first designed on scientific principles the calculations required an undue thickness of wall to prevent tension, which compared unfavorably with the work of ordinary masonry contractors who had built satisfactory stacks for a long time. In order not to be in contradiction with good practice, the custom of allowing tension in the statical calculation was developed, although the originators of this practice knew that such tensions are only imaginary. The straight-line theory was at fault.

The author has endeavored to correct the errors of the straight-line theory, but still persists in the error of assuming imaginary tensions. Tension in a joint indicates the opening of a crack which will have a width at the face of about 0.01 in. when the joints are about 10 ft. apart vertically. What will happen to the dam when the intruding water exerts a pressure in the joint? The great dams constructed by the New York Board of Water Supply have just such a section, and a discovery such as that claimed by the author might cause many sleepless nights to the engineers in charge. While the writer is quite convinced that such water pressure is exerted in the joints of many

* *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1618.

† *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1667; also, Vol. 93 (1929), p. 1250.

dams, he quite admires the judgment of the New York engineers who adopted this profile. It is entirely safe although it has been criticized lately by a number of engineers. Water pressure in joints opened up by the deformation of properly constructed dams is not dangerous because the joint just above it also exerts a downward pressure which is only slightly less than the one below it and the difference of pressure does not produce a great change in the stresses. The writer has shown* that a stress distribution according to a cubic parabola leads to reliable and easily computed results.

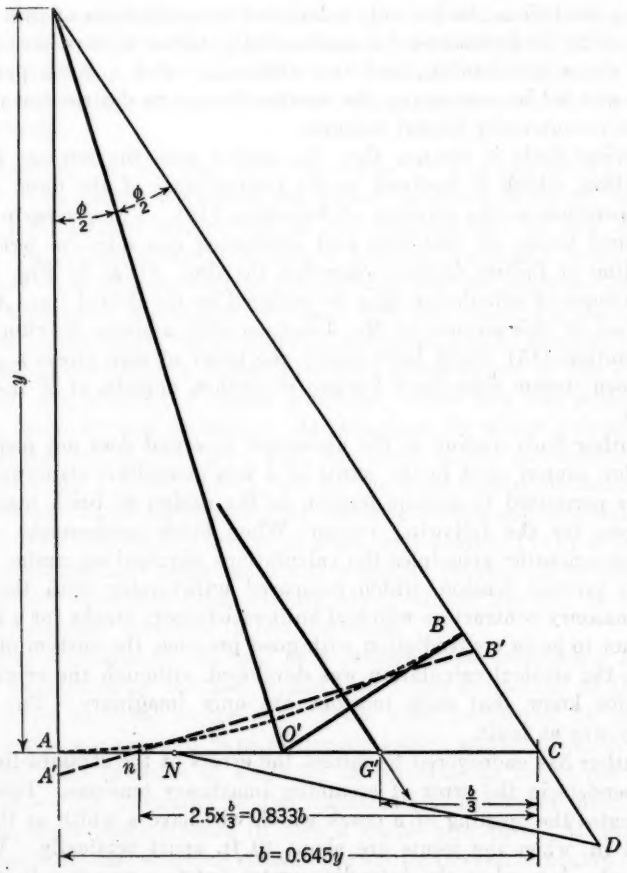

FIG. 15.

Fig. 16 shows the properties of such a parabola and G denotes the center of gravity of all stresses. In a horizontal section such as that chosen by the author, the point, G', in Fig. 16, must be at the intersection of the resultant of all forces with the line, $A\,C$, of Fig. 15. The distance of the point of zero

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1667; also, Vol. 93 (1929), p. 1250.

stress from the down-stream face will be $2.5 \times \frac{b}{3} = 0.833\ b$, provided it is at all permitted to assume such a stress distribution in a horizontal section. Since the vertical load at G' in Fig. 15 is $\frac{1}{2}\ w\ y\ b$, the maximum vertical stress, in pounds per square foot, is, $\frac{\frac{1}{2}\ w\ y\ b}{\frac{3}{4} \times (0.833\ b)} = 0.8\ w\ y$. By a very laborious calculation the author obtains $0.775\ w\ y$; the straight-line theory gives $w\ y$ as maximum vertical stress. An investigation of the section, $AO'B$ (Fig. 15), shows that the stresses parallel to the down-stream face are about $1.17\ w\ y$, in which, y is the depth below the top at Point B; but the point of zero stress is now at N. The failure of the Bouzey Dam, in France, in 1895 shows a line of failure following the line, AND, in Fig. 15. The writer conceives this failure as follows: After tension, in a section, $AO'B$, developed to a point, N, the shear in the line, $N\ D$, became greater than the leached-out mortar of the masonry could stand and the dam sheared off. The shear is a maximum along a line inclined nearly 45° with the principal stress parallel with the down-stream face, and thus the line, $N\ D$, was located. Hence, shear failures are very dangerous to masonry dams as commonly built, or to concrete dams with improper horizontal and vertical joints. These shearing stresses are probably the dangerous ones in high dams of non-uplift section. For dams 700 ft. high they might reach values of 500 lb. per sq. in.

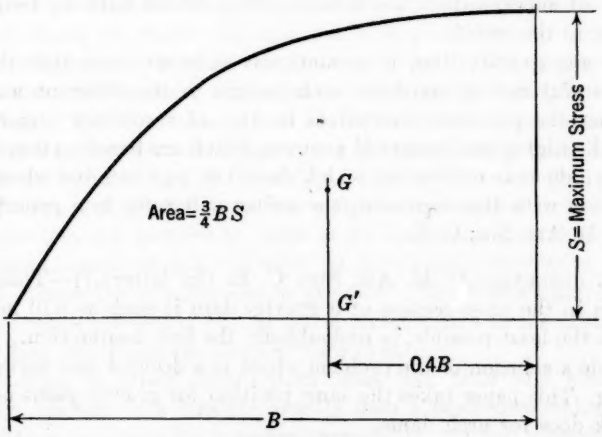

FIG. 16.

When a dam is sheared off at its foundation, or at higher elevations, it will have to move quite a distance before it topples over; consequently, an arched gravity dam must be safer than a straight dam. Most engineers deny the help given to a gravity dam by arch action, because the deflection of the dam as a cantilever is too small to allow of arch action. If, however, through

some error, dangerous shearing stresses should develop, the movement will be large enough to close vertical cracks and open expansion joints, and arch action will prevent complete failure. The danger to a dam without an uplift section is in the foundation. When uplift can be exerted at the base of the dam, that is much greater than the pressure in the joints above it, due to the rock being weakened by the intruding water, as in the St. Francis Dam, or by fissures in the rock below the dam, then failure is likely to occur sooner or later, and for such doubtful foundation conditions a much larger section of dam must be adopted or resort must be had to deep anchorage into the rock near the up-stream face.

JOHANNES SKYTTE,* ASSOC. M. AM. SOC. C. E. (by letter).†—Mathematics and Physics are two useful and much needed tools for the engineer, and Mr. Jakobsen has used these tools in a masterly fashion in finding the stress distribution in gravity dams by the principle of least work. This principle, which can be expressed in the sentence, "Nature is intelligently lazy", appeals to the writer.

Mr. Jakobsen's statement‡ that the sum of n_y in Equation (34)§ and n'_y in Equation (35)§ should be equal to n_y in Equation (27)‖ is correct, but the explanation for the discrepancy is incorrect. The value of n_y in Equation (27) must always be equal to $n_y + n_y'$ (Equations (34) and (35)), provided the assumptions made in expressing n_y as a function of x and y are identical for all three cases. The function for n_y in the present case has been assumed to be an infinite series and has been used with the same number of terms for n_y (Equation (27)), n_y (Equation (34)) and n_y' (Equation (35)). Consequently, by the law of superposition the infinite series should have no bearing on the discrepancy in the result.

Almost any gravity dam is so statically indeterminate that the engineer must be careful not to attribute such weight to the different mathematical theories that the practical viewpoints become of secondary importance. He should let the highly mathematical answers, which are based on certain assumptions, be a guide as to tendencies, which should be kept in mind when designing. In connection with this comment, the writer will refer to a paper¶ by M. H. Gerry, Jr., M. Am. Soc. C. E.

LARS R. JORGENSEN,** M. AM. SOC. C. E. (by letter).††—That the stress distribution in the cross-section of a gravity dam is such as will make the internal work the least possible, is undoubtedly the best assumption. Mr. Jakobsen has made a solution of the problem which is a decided step forward in dam engineering. This paper takes the same position for gravity dams as Professor Cain's work does for arch dams.

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* Asst. Hydr. Engr., Hetch Hetchy Project, San Francisco, Calif.
† Received by the Secretary, October 6, 1930.
‡ *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1625.
§ *Loc. cit.*, p. 1624.
‖ *Loc. cit.*, p. 1621.
¶ "Building Code for Dams," *Western Construction News*, September 10, 1930.
** Cons. Engr., Constant-Angle Arch Dam Co.; Hydro-Elec. Engr., San Francisco, Calif.
†† Received by the Secretary, October 14, 1930.

The curved shape of the stress distribution diagram appeals to the instinct as being more correct than the linear. The inside of a great body should exert greater resistance than the edges. The curved shape of the stress distribution made by the author checks the English tests; there is considerable satisfaction in this. The material in the English tests evidently had a constant value of E whether wet or dry.

The writer has come to the conclusion that for tests, the model ought to be of the same material as the dam itself. Even then it is difficult to reproduce equality. One reason is that the modulus of elasticity for concrete is extremely variable between the wet and the dry faces for sustained load. On a straight gravity dam this fact may not have much influence on the stress distribution, but it will have great influence on the deflection in a down-stream direction.

Mr. Jakobsen refers* to tests made by R. E. Davis, M. Am. Soc. C. E., in connection with the concrete in the Stevenson Creek Dam. The writer regards these tests as most valuable, since they are made with dam construction in view. Professor Davis finds the instantaneous deformation and the plastic flow after, say, 300 days of loading, the two deformations are added together and give an expression for the working modulus of elasticity of the material for sustained load. The interesting and valuable part of the tests is that it is shown that E for sustained load is from 60 to 63% higher for wet than for dry concrete for stresses between 600 and 900 lb. per sq. in.

If the dam body is either entirely dry or entirely water-soaked, the working value of E will be constant to all intents and purposes for the entire dam body, and the stress distribution on the horizontal plane will be, as found by Mr. Jakobsen, in accordance with the theory on the principle of least internal work.

For arch dams the worst case and the most plausible one (and, therefore, the one to reckon with) is when the concrete is wet on the up-stream side and dry on the down-stream side, with a more or less uniform distribution between the two faces, then the value of E of the material is approximately 60% higher along the up-stream face than along the down-stream face.

This large variation in the working value of the modulus of elasticity between the wet and the dry faces will materially affect the stress distribution. It will cause the arch stresses to move in an up-stream direction, so to speak, in the horizontal arch section, relieving the maximum compressive stress at the abutments, where the compression is greatest. The water-soaking effect will relieve the tensile stresses, if any, at the abutments, due to the expansion of the up-stream face relative to the down-stream face.

The change in stress distribution does not alter the arch deflection appreciably, whereas on a straight gravity dam a variable working value of E may not change the stress distribution appreciably, but will materially alter the deflection of the dam in a down-stream direction.

As indicated previously, the assumption of a more or less uniform distribution of the water-soaking effect between the up-stream and down-stream faces is most plausible. This is in line with the ordinary assumption made for the action of uplift.

* *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1614.

When computing maximum arch stresses in accordance with Professor Cain's formulas, it seems best to neglect any help as to better distribution obtainable from the water-soaking effect and the variation in the value of E between the two faces for sustained load, since their values can not be definitely measured; but if that is done, there is no reason why the maximum compression could not be run up to, say, 1 000 lb. per sq. in., provided the engineer insisted upon a 28-day strength of the concrete of 3 000 lb. per sq. in. The 1-year strength of this concrete would be about 5 000 lb. per sq. in. and, therefore, the load stress would be carried with a margin of five.

It is not possible to design really high dams economically using maximum stresses as low as, or lower than, 600 lb. per sq. in., and since the maximum stresses only exist in parts of the dam, it should be entirely practical to make these parts of a better grade of concrete. Such parts would be near the abutments and near the bottom of high dams. Since E is not constant anyway and is least on the down-stream side, it seems perfectly logical to introduce a higher value where this is mostly effective in the form of richer concrete. With good material and good workmanship a 28-day strength of 3 000 lb. per sq. in. can be obtained in most cases, with but little more than 1 bbl. of cement per yd. of concrete.

The concrete in the Diablo Dam, on the Skagit River, in Washington, which has just been completed, has a 28-day strength of 4 200 lb. per sq. in. This strength was insisted on, and it took 5 sacks of cement per cu. yd. of concrete to produce it. In the writer's opinion this strength was unnecessarily high since the maximum stresses in the dam were moderate, but the engineers for the City of Seattle decided it was worth while. Concrete with a 28-day strength of 3 000 lb. per sq. in. can be depended upon for good lasting qualities; high dams should be built of such material only. Such dams would have an ample safety factor even with an apparent compression of 1 000 lb. per sq. in. in some of their parts.

With Mr. Jakobsen's assumptions of $k = 0.645$ at 100 ft. below the crest and no uplift, it is apparent from Table 1* and Fig. 4,* that in order to eliminate tension in the up-stream face entirely, k would have to be increased to about 0.70. This is of rather great importance. Since uplift is an unknown factor and since the most positive way of taking care of it is by adding to the weight of the dam, to the base width, or to both, a further change of k would be necessary.

The writer feels that nothing less than $k = 0.75$ should be adopted for a triangular section of a gravity dam. For very high dams, and for dams in which it is necessary to assume considerable uplift pressure, k will be still greater, but a minimum of $k = 0.75$ ought to be established by the Engineering Profession, for dams of moderate height on good foundations. Mr. Jakobsen has shown† that for full uplift it is necessary to make $k = 0.844$.

* *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1621.
† *Loc. cit.*, p. 1626.